

DOCUMENT RESUME

ED 218 092

SE 037 886

AUTHOR Maher, Carolyn A.; And Others
TITLE Variables Associated with Problem-Solving
Performance: The Rutgers University-Middlesex Public
Schools Math Project.
PUB DATE Mar 82
NOTE 126p.; Paper presented at the Annual Meeting of the
American Educational Research Association (New York,
NY, March 18-23, 1982).
EDRS PRICE MF01/PC06 Plus Postage.
DESCRIPTORS Cognitive Development; *Cognitive Processes;
Educational Research; *Grade 8; *Grade 9; Learning
Theories; Longitudinal Studies; Mathematics
Education; Mathematics Instruction; *Middle Schools;
*Problem Solving; Secondary Education; *Secondary
School Mathematics
IDENTIFIERS *Mathematics Education Research

ABSTRACT

This study used 152 grade eight students ranging in age from 12 to 14 years from a suburban middle school in central New Jersey. The investigations gave particular attention to the identification of variables that represent understanding of fundamental mathematical structures. The first year of the research took place during the 1978-1979 school year. The population for the second year of the study consisted of 123 ninth-grade students who were eighth-grade subjects the first year. The research is viewed as contributing to the operative memory theory linking mnemonic level to the operational level of a subject, and results are termed as consistent with the view that as intelligence develops changes may occur in the way events are remembered. (MP)

* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *

ED218092

Variables Associated with Problem-Solving Performance: The
Rutgers University--Middlesex Public Schools Math Project

Carolyn A. Maher, John J. O'Brien, Lucille Heller,
Judy F. Smith, Bruce Normandia and Frank DeStefano
Rutgers University

"PERMISSION TO REPRODUCE THIS
MATERIAL HAS BEEN GRANTED BY

Carolyn A. Maher

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)."

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)

☒ This document has been reproduced as
received from the person or organization
originating it.
☐ Minor changes have been made to improve
reproduction quality.

• Points of view or opinions stated in this docu-
ment do not necessarily represent official NIE
position or policy.

Paper presented to the American Educational Research Association,
New York, March 23, 1982

SE 037 886

The investigations reported here respond to the view that any performance and specifically performance in mathematics is the outcome of the joint interaction of many variables. In an effort to understand better the large, multifaceted, complex situation that characterizes mathematical performance, an extended program of coordinated research studies designed to include many measurements of the performance of the same population from various points of view and at various points in time over a two-year period was undertaken. Particular attention was given to the identification of variables that represent understanding of fundamental mathematical structures. Furthermore, longitudinal studies of the same population made possible the monitoring of changes in performance as influenced by developmental changes. The study of durability of learning, scheme stabilization and the like now became possible as well as the identification of those variables significantly associated with successful computation, concept attainment, and word problem solving performance in mathematics.

"Standard" form problems in mathematics may be solved successfully by students algorithmically while variations in form present difficulty. On the premise that a broad base of knowledge reduces the difficulty, an investigation of the presence of such a base in

successful problem-solvers was undertaken. Our view is that successful problem solving requires that the mathematical knowledge of students enables them to recognize the structure of non-standard problems at the relevant level of complexity in order to express it in a form capable of being processed to a solution.

If the recognition of the structure involved in a problem is the condition necessary for insightful solution, it may be hypothesized that the characteristics of successful performance in mathematics must include the aptitude to call forth from the structures within the store of those available to the student the structures that are isomorphic with those involved in the problem.

Structures selected for study were those commonly involved in tests of problem-solving administered to the population under study, although ordinarily at somewhat lower levels of complexity.

Preliminary work had established relationships between certain structures and particular elements of mathematical performance. Maher (1980) demonstrated strong relationships between the structures of group properties and proportional logic to computation. Maher and O'Brien (1981 a) demonstrated similar relationships between the structures of permutation attainment and proportional logic to problem solving test performance.

Silver (1979) reported that "mathematically capable students have the ability to grasp the mathematical structure of a problem and to generalize to problems of similar structure (p. 208)." Piaget and Inhelder (1973) disclosed the role of memory in summoning appropriate structures to the solution of a problem.

Hence, one of the objectives of the coordinated research program was to examine the relationship of students' performance on tasks involving several structural variables to performance on tests of problem solving. In particular, two investigations reported here provide models which relate the associations among significant structural variables and problem-solving test performance. Considering a fixed number of subjects and a multivariate situation for purposes of analysis, each report limits the discussion to three structural variables.

Method

The students consisted of 152 eighth graders ranging in age from 12 to 14 years from the Middlesex Public Schools, a suburban middle school in central New Jersey. The average IQ was 100 (Otis Lennon).

All clinical tasks were administered individually to students by clinicians who received extensive training. Earlier preparation included the observation of an interviewer's testing of a subject by at least two other

clinicians. Each clinician independently recorded the subject's performance on the task and final ratings and responses were compared at the conclusion of each task. A clinician was permitted to participate in the study when there was agreement in the recording and rating of the subject's responses among all the clinicians administering the task. All interviews were conducted according to protocol specifications. For some tasks, audio-tapes of the interviews were made.

All written tests were administered in the intact classes of the students by the regular classroom teacher.

The following structural variables were assessed at the time intervals specified.

1. Understanding of a proportional logic structure (PLS) was determined by evaluation of a subject's performance on a clinically administered balance beam task which, according to Inhelder and Piaget (1958), requires formal thought for successful completion. The task involves balancing a beam with two weights by taking into account the distance of each from the fulcrum. Thus an understanding of the proportionality between weight and distance was necessary for success (see Appendix A). Successful subjects were classified as formal and unsuccessful as nonformal according to criteria described by Inhelder and Piaget (1958).

Students classified as belonging to Stages IIA and IIB were considered to lack the structure of proportional

logic and were classified as nonformal while those classified as belonging to Stages IIIA and IIIB, demonstrating the understanding of coordination of weights and distances, were classified as formal.

Each balance beam rating was made by the validated clinicians administering the task at the conclusion of each interview and later all forms were reviewed by a committee of researchers. In instances where there was doubt or disagreement, the form and tape were reviewed by at least two clinicians for a final decision. In all such cases a final determination was reached by a consensus of the reviewing clinicians. Inter-rater reliability was .96 on the balance beam task.

It is of interest to note that the topic of proportions is generally included in mathematics programs beginning in the sixth-grade. Also, exercises calling for these schemes were found in all commercially published tests for mathematics achievement. Hence, the inclusion of proportional logic as a structural variable for consideration seemed appropriate.

The balance beam task was administered at the onset of the study, again after seven months, and for those students not yet successful, again at seventeen months. For the problem solving reports, combined initial and seven-month task results were used.

2. Understanding the structure of the properties of group (GPS) was determined by a task that was clinically administered to each student. The task, developed by the Rutgers Research Group, involved the manipulation of a boy and girl doll. The elements of the set were turns of either 0 or 180 degrees (moves/no moves) of the dolls; only boy turns, only girl turns, both turn, nobody turns. The operation applied to the moves was "followed by". The task consisted of two parts: a training session in which subjects were given as much time as necessary to demonstrate understanding of the nature of the commands and the operation by correctly completing a table requiring all possible outcomes of the "followed by" operation; and an evaluation component in which understanding of the structure of each of the properties--closure, commutativity, identity, inverse and associativity--was measured by recording a student's response to a series of questions provided in the protocol (see Appendix B). Subjects were provided with the figures and a set of command cards on which the elements of the set were recorded and encouraged to use them as deemed necessary throughout the task. Students were judged to be successful (high) if they spontaneously or after one inductive cycle successfully identified each of the five properties or required two inductive cycles for only one of the five properties. Otherwise, they were classified as unsuccessful (low). The task was administered initially and for those

classified as unsuccessful, again one year later. The individual responses to the group properties structure (GPS) task were scored independently by two clinicians. All ratings were compared in committee with a third clinician and differences were resolved by a consensus of the reviewing researchers. Inter-rater reliability was .96 for the group properties structure (GPS) task.

It is contended that the structure of the properties of a group provides an organization that encompasses many basic concepts of mathematics. The presence of such an organization makes possible, for example, the recognition of the identity in the problem $\frac{3}{4} \times \frac{5}{\square} = \frac{15}{20}$ to generate an equivalent fraction providing for those students who recognize it, an additional opportunity for a successful solution compared with those who mechanically employ a division rule.

Also, according to Bruner (1961), such an organization provides a basis for understanding the process of solving a linear equation.

3. Cognitive style structure (CSS) was measured by a subject's performance on the Group Embedded Figures Test published by Consulting Psychological Press, (1971). Cognitive or perceptual style is, according to Witkin, et. al. (1971) an approach which a subject brings to an array

of situations of certain structure. Subjects tending toward field independence perceive parts of a field as separate from the whole while subjects tending toward field dependence perceive parts as embedded within the whole. Lester (1980) reports that field independence tends to be related to problem solving ability. Good problem solvers tend to be field independent, tend to apply a wider range of heuristics, and are more willing to vary their approaches (Blake, 1977).

Students were judged to be tending toward field independence (high) if they scored 13 or more on the cognitive style structure (CSS) task. Students with scores less than 13 were judged to be tending toward field dependence (low). The range of scores was 0 to 18. The cognitive style structure task was scored independently by two researchers. Comparison of the objective scores revealed no inconsistencies.

4. Remembrance of a combinatorial logic structure (RCS) was determined by evaluating a subject's performance on a clinically administered memory task immediately, at one week, at six months and at 18 months. The mnemonic task required the remembrance of an arrangement of three geometric forms (circle, triangle, and square) taken two at a time (see Appendix C). According to Piaget and Inhelder (1973) remembrance of the permutation over time indicates the presence of operative combinatorial schemes that manifest formal thought operations for this structure.

At the initial presentation each clinician presented to a subject a 28 cm by 22 cm card containing a picture of the geometric figures. The subject was then asked for a brief description of the picture. The subject was also asked if the arrangement was "mixed up" or if there was some "order to it". Lastly, the subject was asked to examine the card for as long as necessary and told that a memory drawing of the figure would be requested later. Upon completion of the examination of the drawing, the card was removed and a memory drawing requested. During the six and eighteen-month interviews a discussion to elicit the memory drawing was made, reminding subjects that they were shown a card containing some geometric figures earlier. Subjects were also reminded that they were asked whether the geometric figures were "mixed up" or if there was some "order to it". No specific information which identifies the geometric figures or their arrangement was given to the subjects. Also, each subject was given a task assessing recognition of the arrangement requiring the selection of one drawing from among four (see Appendix C) which most closely resembles the one shown earlier. The subject was then asked whether or not the drawing selected was an exact copy of the earlier one.

According to the operative view of intelligence, there is a functional unity among the levels of

intelligence in that earlier stages prepare for later ones and later stages enrich and generalize earlier ones. In this sense recognition is regarded as a lower order hierarchic form of memory.

The ratings of the memory drawings were made based on the same system of levels and sublevels described by Piaget and Inhelder (1973). Each memory drawing was scored independently by two clinicians and all ratings were compared in committee with a third clinician. No differences were found in immediate memory assessments. Inter-rater reliability was .98 for one-week recall, .95 for six-month recall and .96 for eighteen-month recall. The recognition test was objectively rated by a committee of researchers and checked for accuracy and clerical mistakes. No errors were found in the scoring of this task.

5. Mathematical problem solving test performance (MPS) was measured by the score achieved on the Advanced Metropolitan Problem Solving Test, Form F, administered at approximately the same time as the second balance beam task. The tests were machine scored by the test publishers and problem solving scores were categorized as high (at or above grade level--a score greater than or equal to 8.5) or low (below grade level--a score less than 8.5).

The content of the 35 item multiple choice word problem test included (a) 4 one-step addition, subtraction,

multiplication and division problems, (b) 9 fraction and decimal problems, (c) 1 general solution problem, (d) 4 multiple-step problems, (e) 3 geometry problems, (f) 3 ratio problems, (g) 2 percent problems, (h) 1 negative integer problem, (i) 3 chart reading problems and (j) 5 number sentence problems.

The test is typical of other commercially available ones in terms of the range of topics and the level of difficulty for measuring problem solving in school mathematics. Despite the apparent limitation in failing to provide information on the process by which a solution is reached, these instruments nevertheless are used widely for such purposes as student placement into mathematics courses, student identification for remedial or enrichment programs, and grouping within existing mathematics subject areas. Because of such pervasive use in schools, an exploration of the relationship of the knowledge base of subjects in terms of the structural variables identified as related to success on the MPS seemed worthy of study.

Report 1

The foregoing analysis focuses on the relationship of the three structural variables, proportional logic structure (PLS), remembrance of combinatorial logic structures--six-month recall and recognition (RCS) and cognitive style structure (CSS) to mathematical problem solving test performance (MPS).

The correlations between the variables used are presented in Table 1.

Insert Table 1 about here

Table 1 contains the Yule's Q correlation matrix for the following assessments: proportional logic relating to formal thought, recall and recognition memory relating to formal thought, cognitive style relating to field independence and problem solving success. While all assessments are significantly correlated to problem solving success ($p < .001$), they are also significantly correlated with each other. Yule's Q provides a measure of the strength of the individual pairwise associations.

Although the focus of this investigation was on the relationship of the structural variables to mathematical problem solving test performance, the question of the significance of students' IQ measure (Short Term Test of Academic Aptitude, McGraw-Hill) to problem solving was considered. IQ is significantly correlated to problem solving (.41) and to cognitive style (.41). However, the IQ measure did not significantly improve the model used

to examine the association among the three structural variables and mathematical problem solving test performance.

Figure 1 displays in bar graph form the results of the assessments obtained on Piagetian proportional reasoning, six-month mnemonic level, cognitive style and their relation to problem solving level. Loglinear modeling and a conditional and path analysis were employed to evaluate the model.

Insert Figure 1 about here

Fifty-one percent of the 152 subjects in the population scored at or above grade level on problem solving. The shaded region represents the proportion of students for each assessment who were at or above grade level in problem solving performance. Forty-six percent of the subjects were assessed as formal on proportional reasoning and 70 percent of those subjects were rated high on problem solving performance. Fifty-four percent of the subjects were assessed as nonformal on proportional reasoning and 35 percent of those subjects were rated high on problem solving performance. This represents a 35 percent increase of scoring at or above grade level on problem solving for subjects assessed as formal than those assessed as nonformal on the balance beam task. For the logical memory assessments the problem solving success rates are 79, 55, 32 percent for recall, recognition and no recognition respectively. Forty-seven percent of

the students were assessed as field independent on cognitive style and 74 percent of those subjects were rated high on problem solving performance. Of the remaining 53 percent of the subjects assessed as field dependent only 31 percent scored at or above grade level on problem solving performance reflecting a 44 percent difference in the two categories of students.

The results of the information in Figure 1 suggest the importance of each of the three structural variables to mathematical problem solving test performance.

In an earlier study, Maher and O'Brien (1980) found a significant correlation between cognitive level relating to formal thought and logical memory. The relationship of problem solving success to students joint cognitive and long-term mnemonic levels is shown in Figure 2.

Insert Figure 2 about here

Only three of the students assessed as nonformal had recall memory, a result consistent with theoretical expectations concerning proportional logic levels relating to formal thought and combinatorial memory structures (Piaget and Inhelder, 1973). Hence, for nonformal students, recall and recognition memory categories were combined and compared with students who did not recognize.

For students assessed as formal, there was no significant difference (.64 vs. .66) in problem solving success for the recognition and no recognition categories. Hence, for formal students the recall and recognition/no recognition categories were compared. Figure 2 illustrates that 79 percent of formal students who recalled scored at or above grade level on problem solving and 20 percent of the nonformal students without recognition memory were successful.

The relationship of problem solving success to student's joint proportional reasoning and cognitive style ratings is shown in Figure 3.

Insert Figure 3 about here

For students assessed as formal on proportional logic structure, 81 percent of those classified as tending toward field independence scored at or above grade level on problem solving while 49 percent of those classified as tending toward field dependence were successful on problem solving. For students assessed as nonformal on proportional logic structure, 60 percent of those classified as field independent scored at or above grade level on problem solving while 23 percent of those classified as field dependent were successful on problem solving.

Figure 4 illustrates the impact on problem solving success of the three assessments: proportional reasoning

structure, combinatorial memory structures and cognitive style structure.

Insert Figure 4 about here

For students rated as formal on proportional logic structure, field independent on cognitive style and having six-month recall memory, eighty-eight percent scored at or above grade level on problem solving and for students rated as nonformal on proportional logic structure, field dependent on cognitive style and without recognition memory, eleven percent were successful.

Students assessed as formal and field independent but not having recall memory show a tendency (76 percent) for success on problem solving. Students assessed as nonformal, field independent and having recall or recognition memory (73 percent) also had a tendency for success on problem solving.

Generally, students rated as high on any one assessment have an above average proportion of success on the problem solving test when compared to the population mean of 51 percent. The proportion of students experiencing success on problem solving increases or remains the same for students rated high on any two assessments and decreases for students, rated low on the corresponding assessments. A similar pattern is found when proportional reasoning, six-month mnemonic level

and cognitive style are viewed jointly with respect to success on problem solving.

The preceding analysis has revealed significant correlations among all pairs of the variables under study as well as their individual and joint importance in determining problem solving success. A further objective of this study is the investigation of the relationship among all of the variables in a multivariate context. These relationships may be summarized in the diagram presented in Figure 5.

Insert Figure 5 about here

The purpose of the following analysis is to examine the magnitude of the joint associations among proportional logic structure (A), six-month memory structures (B), cognitive style structure (C) and problem solving (D). The associations among the variables are described via the relative odds of a high score on one of these assessments conditioned on the others. These associations are derived from loglinear model estimates of the expected cell counts of the data in Table 2. The loglinear model estimates in Table 2 used the observed cell counts as input data and were derived using the maximum likelihood estimation procedures contained in the BMDP computing package (Brown, 1979).

Insert Table 2 about here

The goodness of fit of a loglinear model for predicting the cell counts of a cross-classification table is defined as the percentage of variation explained by the loglinear model relative to the independence model. The loglinear model described by Equation 1 explains 96 percent of the variability of the data in Table 2 ($R^2 = .96$) when compared to the independence model.

$$\begin{aligned} \log m_{ijkl} = & \lambda + \lambda_i^A + \lambda_j^B + \lambda_k^C + \lambda_l^D \\ & + \lambda_{ij}^{AB} + \lambda_{ik}^{AC} + \lambda_{il}^{AD} + \lambda_{jk}^{BC} + \lambda_{jl}^{BD} + \lambda_{kl}^{CD} \\ & + \lambda_{ijk}^{ABC} + \lambda_{ijl}^{ABD} \end{aligned} \quad (1)$$

Odds for a given event (successful problem solving for example) conditioned on the other variables in the table is the ratio of the number of high to low rated subjects within a cell of the contingency table. The odds in favor of successful problem solving, for example, are dependent only on those variables in the model interacting with it-- in this case all three structure variables. Ninety-five percent confidence interval estimates of the odds of high problem solving as a function of a subject's proportional logic structure, six-month memory structures, and cognitive style structure are contained in Table 3.

Insert Table 3 about here

Ninety-five percent confidence interval estimates for the relative odds of high problem solving conditioned on proportional logic structure, six-month memory structures and cognitive style structure are found in Table 4.

Insert Table 4 about here

Only the conditional and relative odds whose confidence intervals do not contain one are considered significant and reported in the summary path analysis in Figure 6. The information in Table 4 shows for subjects who did not have recognition memory, a subject with a formal assessment on proportional reasoning has 7.74 times the odds of success on problem solving as a subject with a nonformal assessment. Students with a nonformal assessment, who were successful on the recognition memory task had 4.26 times the odds of a high problem solving rating as those who were not successful. It should also be noted that a field independent subject had 5.25 times the odds of success on problem solving as a field dependent subject regardless of the outcomes of the other assessments.

The results of the conditional relative odds analysis are presented in the path analysis of Figure 6.

Insert Figure 6 about here

All pairs of assessments that are conditionally correlated are connected with paths. The path measures

are the conditional relative odds values discussed earlier. In general the path weights are symmetric and this is indicated by the double headed paths in Figure 6. Path weights such as AC are conditioned on a third variable (B) due to three-way interactions contained in the "best fitting" loglinear model. Specifically, subjects successful on the recognition memory task who had a formal cognitive assessment had 5.59 times the odds of a field independent cognitive style rating as subjects classified as nonformal. Similarly the BC path shows that subjects assessed as formal (A+), who were successful on the recall recognition memory task had 6.27 times the odds of field independent rating as subjects who were not successful. The AB path illustrates that a subject high on problem solving (D+) and cognitive style (C+) who was assessed as having a formal assessment on proportional logic structure had 4.78 times the odds of having recall memory as a nonformal subject. The AD, BD, and CD paths mentioned earlier in the discussion of relative odds shown in Table 4 are also included in the path analysis of Figure 6.

Report 2

This study explored the associations among three structural variables and mathematical word problem solving. The structural variables and the dichotomous levels of assessment were:

- (A) Group Properties Structure (GPS)--understanding, not understanding
- (B) Proportional Logic Structure (PLS)--formal, nonformal
- (C) Cognitive Style Structure (CSS)--tending toward field independence, toward field dependence

Performance in mathematical word problem solving (MPS) was evaluated by grade level; at or above versus below grade level. The measurements of all four variables were available for 113 of the 152 subjects participating in the Middlesex Project.

In order to examine the strength of the individual pairwise associations among the structural variables and problem solving, the correlation matrix given in Table 5 was generated.

Insert Table 5 about here

This table shows the measures of the group properties structure, proportional logic structure and cognitive style structure are significantly correlated to success in

mathematical word problem solving at the 0.05 level; however, the group properties and cognitive style measures are the only structural variables correlated at the same level.

The relationships among these variables were studied further using bar graphs and loglinear modeling. In addition, a path analysis was included to facilitate the interpretation of the loglinear model.

Of the 113 subjects included in this analysis, 44 percent were successful in problem solving as indicated by scoring at or above grade level. The relationship of this success in problem solving to each of the structural variables is illustrated in Figure 7.

Insert Figure 7 about here

The shaded region of the bar graphs in Figure 7 represents the proportion of students who were successful in problem solving as a function of the various categories of the structural variables. It can be observed that of the 52 percent of the subjects rated high on the GPS, 57 percent of them scored at or above grade level in problem solving as compared to 31 percent of those rated low on the GPS. Thus, there is a 26 percent difference in the rate of successful problem solving between those subjects rated high on the group properties structure and those at the low level.

Of the 38 percent of the subjects who were at the

formal level in proportional reasoning on the balance beam task, 57 percent were successful in problem solving as compared to 37 percent of those who were nonformal on this task. Of the 40 percent of the subjects who were assessed as tending toward field independence, 65 percent were successful in problem solving compared with 32 percent of those who tended toward field dependence.

From Figure 7 it can be noted that if students ranked high on each of the assessments the proportion achieving success in problem solving exceeded the comparable proportion of those at the low level by 20 percent. The largest difference, greater than 30 percent, was attributable to cognitive style.

The importance of measures of cognitive style structure and group properties structure is examined in Figure 8.

Insert Figure 8 about here

Figure 8 shows that 75 percent of the subjects who were rated high on group properties structure task and who tended toward field independence scored at or above grade level in problem solving while only 43 percent of those with a high assessment on the GPS who tended toward field dependence scored high on problem solving. Fifty percent of those who scored low on the group properties task and tended toward field independence were successful in problem solving compared to 20 percent

of those who scored low on the GPS and tended toward field dependence.

Insert Figure 9 about here

Figure 9 shows the joint relationships of the assessments of the group properties structure and proportional logic structure. Of the students who were rated high on the GPS and formal on the proportional logic structures task, 68 percent were successful in problem solving compared to 49 percent of those who were rated high on the GPS and were non-formal on proportional reasoning. Of those scoring low on the GPS and formal on the PLS, the percentage successful in problem solving was 42 percent compared to 25 percent for those who rated low on the GPS and nonformal on the PLS task.

The relationship of all three structural variables to problem solving is shown in Figure 10.

Insert Figure 10 about here

Of the subjects who ranked high on the assessments of all three structural variables, 83 percent were at or above grade level on problem solving. In contrast, 16 percent of those who were uniformly low on all three measures were successful in problem solving.

The data used in this study are described by level in

the four-way classification table given by Table 6.

Insert Table 6 about here

This table shows the number of observed and expected frequencies for each cell. The expected values were derived from a loglinear model which represents the most parsimonious model for the experimental data. The model given by Equation 2 was obtained using maximum likelihood procedures described by Haberman (1978) and the stepping and screening procedures of Brown (1979).

$$\log m_{ijkl} = \lambda + \lambda_i^A + \lambda_j^B + \lambda_k^C + \lambda_l^D + \lambda_{il}^{AD} + \lambda_{jl}^{BD} + \lambda_{kl}^{CD} \quad (2)$$

This model explains 84 percent of the variation not accounted for by the model in which problem solving is assumed to be independent of the GPS, PLS and CSS. This model was used to derive the conditional odds in favor of achieving at or above grade level performance on the problem solving test shown in Table 7.

Insert Table 7 about here

Here, odds has the usual definition--the ratio of the number of subjects achieving at or above grade level on the problem solving test to those below grade level. Ninety-five percent confidence interval estimates of the conditional odds are also shown in Table 7.

For a specific variable, say the GPS, the high GPS to low GPS relative conditional odds of successful problem solving is defined as the ratio of the corresponding conditional success odds. Thus, in Table 8, the relative odds for subjects rated as understanding the group properties structure over those lacking this understanding are determined by a ratio in which the levels of the other two variables, measures of the proportional logic structure and cognitive style structure are fixed.

Insert Table 8 about here

For those students displaying an understanding of the group properties structure, Table 8 shows that regardless of proportional reasoning level and cognitive style, the odds in favor of at or above grade level performance on the mathematical word problem solving are three times those of a subject not understanding the group properties structures. A formal subject has comparable odds of 2.2 over nonformal subjects and for subjects tending toward field independence, the odds are four times that of subjects tending toward field dependence.

The path analysis illustrating the association among the three structural variables and problem solving is shown in Figure 11.

Insert Figure 11 about here

The relative conditional odds listed in Table 8 are used to determine the path weights and the doubled headed paths indicate the symmetry of these weights. The absence of a connecting path between two variables indicates the conditional independence of these measures. Thus, Figure 11 shows the assessments of the structural variables are each related to problem solving; however, these structural variables are mutually independent implying that a high rating on one structural variable does not suggest that any other variable is also likely to be high. The path weights provided information on the odds in favor of success in problem solving for all combinations of the levels of assessment on each measure. For example, from Figure 11 the odds for success on problem solving for subjects ranking high on all three assessments are seen to be 26.4 ($4 \times 3 \times 2.2$) times that of subjects rated low on all three. For subjects ranking high on the GPS and low on PLS, the odds in favor of successful problem solving performance are $1.8 \left(4 \times \frac{1}{2.2}\right)$ times those for subjects low on the GPS and formal in proportional reasoning.

In summary, this analysis highlights the conditional independence among the three structural variables and suggests that success in problem solving assessed by the test used in this study requires that the student rank high on all three structural measures.

Conclusions

These reports complement each other in the finding that students exhibiting understanding of the structures studied perform better on mathematical problem solving tests than students who did not. We speculate that success on these tasks indicated the presence of more complex levels of structural organization of mathematical knowledge. It seems reasonable to expect a student to have a greater chance of success in solving problems when the available structures are more highly developed. For example, the student with formal proportional logic schemes should have greater likelihood of solving a proportion problem that is capable of solution with additive schemes than a non-formal student who's schemes are less fully developed.

Current efforts are being directed towards an examination of the relationship of levels of understanding of the structures examined in this report to success with problems more directly related to those structures.

The Longitudinal Studies

The general objectives of the investigations that took place over the two-year period were (1) to monitor change over time with respect to measurements of the structural variables, (2) to examine the relationship of students' performance on tasks involving certain structural variables to learning given mathematics concepts and (3) to extend from six to eighteen months an examination of the relationships observed in the six month studies.

Sample

For the first year of the two-year study, the subjects consisted of 152 eighth-grade students of the Middlesex New Jersey Public School System (1978-1979). Of that group, 137 ninth graders were enrolled in the high school in 1980-1981, representing 90 percent of all subjects from the original eighth-grade group.

Another group of eighth-grade students were studied in 1979-1980. These 141 subjects participated in a separate study and the details are not provided in this report.

Method

The following variables were clinically assessed at various times initially and again after specified time

intervals:¹

1. Understanding of a proportional logic structure
(initially, 7-months, 17-months²)
2. Understanding the structure of the properties of
a group (Initially, 12-months³)
3. Remembrance of a combinatorial logic structure
(initially, one-week, 6-months, 18-months)
4. Remembrance of a horizontal/vertical spatial struc-
ture (initially, one-week, 6-months, 18-months)
5. Operational assessment of horizontal/vertical
spatial structure (initially, 6-months,
18-months)

The following variables were measured with paper and pencil tests initially:

1. Cognitive style structure (field dependence/
independence)
2. Spatial abilities
 - a. Paper folding
 - b. Card rotations

¹For the longitudinal assessment all subjects previously classified as low were again tested. Only a sample of students who were successful were retested as a check for consistency in performance.

²Of the 62 students formerly classified as successful, nine were retested.

³Of the 48 students formerly classified as successful, eight were retested.

3. Short term memory
 - a. Digit span
 - b. Associative memory
 - c. Visual memory
4. Understanding of the structure of an Abelian group
5. Reading achievement (Metropolitan-1980; Stanford-1981)
6. Mathematics achievement (Metropolitan-1980; Stanford-1981)

Mathematics achievement on New Jersey State Minimum Basic Skills tests and IQ (Short Term Test of Academic Aptitude, McGraw-Hill) scores were also available.

The coordinated research sought to obtain measurements of performance with respect to individual variables on learning as well as the impact of various combinations of them. Considering a fixed number of subjects and a multi-variate situation, some limitations of the number of variables became necessary for purposes of analysis. To accommodate statistical analyses of multivariate discrete data, loglinear modeling was employed. Applications of conditional odds and path analyses techniques made possible an assessment of theoretical models.

Data analysis for the longitudinal investigations of the relationships of the structural variables to mathematical test performance is still in progress. The following report provides a longitudinal extension of an earlier study by

Maier and O'Brien (1981 b) which demonstrates the influence of operative schemes on corresponding mnemonic performance providing further support for the Piagetian operative view of memory.

The 1981 study was designed to test the Piagetian theory that distinct stages in the organization of memory relating to horizontality correspond to successive stages in operative level. Maier and O'Brien reported results which are consistent with the view presented by Piaget and Inhelder (1973). Specifically, they found that immediate recall on the drawing of an inclined bottle with liquid was dependent on an operative assessment related to horizontality given at approximately the same time and on a six-month memory assessment of the inclined bottle (see Appendix D). Six-month recall was dependent on the initial operative assessment, immediate recall, and a six-month operative assessment relating to horizontality.

Objectives

The study was designed (1) to extend from 6 to 18 months an examination of the relationship between Piagetian operative level and memory level relating to horizontality for 13 to 15 year old subjects, and (2) to describe the association between operative and mnemonic levels via a log-linear and path analysis model.

Sample

The population consisted of 123 ninth-grade students from the Middlesex New Jersey Public School System (1979-1980).

Method

Initial operative level was determined by evaluation of a subject's performance on a clinically administered angle measurement task, described by Piaget, Inhelder and Szeminska (1960) (see Appendix E). The six-month operative assessment (see Appendix E) was based on a procedure used by Piaget and Inhelder (1973) that required the subject to state whether the level of the liquid in the drawings was possible and if so, why. Each of the drawings represented a given operational stage according to Piaget. Both operative assessment tasks require a mastery of rectangular coordination for successful completion (Piaget, Inhelder, Szeminska, 1960; Piaget and Inhelder, 1973). However, Maher and O'Brien (1981 b) found that these tasks were only weakly correlated ($r=.15$, $p<.10$) and interpreted this as an indication that these tasks tap different as well as common aspects of the same schemes or operativity in general. Other explanations of this phenomena include the possibility of instability in the construction of the schemes of horizontality in some subjects (Maher and O'Brien, 1981 b).

Results

The association between operative (six-month and eighteen-month) and the corresponding spatial memory assessments are described via statistical models of the odds of correct recall and presented graphically with a path analysis model. Estimates of conditional odds and odds ratios are based upon a loglinear model analysis of the cross-classification table consisting of the number of experimental subjects within each of the memory and operative assessment categories. Maximum likelihood estimation procedures as described by Haberman (1978) were used. The goodness of fit of a memory-operative assessment log-linear correlation model is reported as the percentage of unexplained variation of the corresponding memory-operative assessment independence model which is accounted for by the association between the memory and operative assessments. The log-linear model used for these data explains 91 percent of the variability ($R^2 = .91$).

The analysis revealed significant correlations between operative assessments (six-months and eighteen-months) and memory assessments (six-months and eighteen-months). Subjects assessed as operative at six-months have 6.26 times the odds of correct memory at six-months as those who were assessed as nonoperative. Subjects assessed as operative at eighteen-months have 5.71 times the odds of correct

memory at eighteen-months as those who were not. Significant correlations were found between the operative assessments as well as between the memory assessments. Subjects who had correct memory at six-months and were assessed as operative at eighteen-months had a 3.3 to 1 odds of correct memory at eighteen-months, 15.3 times that of subjects who did not have correct memory at six-months and were nonoperative at eighteen-months. Subjects assessed as operative at both six and eighteen-months had 2.5 to 1 odds of correct memory at eighteen-months, eight times that of subjects assessed as nonoperative at six and eighteen-months.

The data suggest trends of correct memory at eighteen-months paralleling changes in operative level. For example, subjects assessed as nonoperative at six-months and operative at eighteen-months had four times the odds of correct recall at eighteen-months than subjects nonoperative at six and eighteen months. The results are consistent within the Piagetian framework.

Educational Importance

This research while contributing to the operative memory theory linking mnemonic level to the operative level of a subject, also demonstrates the durability of that scheme influence in memory over an eighteen-month period. The results are consistent with the view that as the intelligence develops changes may occur in the way events are

remembered. Furthermore, support for the hypothesized relationship between operative and mnemonic levels was obtained over a longer time period than hitherto has been reported.

Acknowledgement

The research reported herein represents the combined efforts of members from the Rutgers Research Group under the direction of Carolyn A. Maher. The development of protocol and the like, training of clinicians, and the conducting of interviews required substantial contributions from many people. Particular responsibilities were assumed as follows: John J. O'Brien directed the memory and problem solving aspects of the studies; Lucille H. Heller, the cognitive style and problem solving; Frank V. DeStefano, the group properties; Bruce Normandia, the spatial measures; and Judy Smith, the longitudinal aspect.

We express our appreciation to Warren Crown and Richard DeLisi for their helpful suggestions throughout the study.

We are especially indebted to James A. Maher for his guidance with data analysis.

We wish also to acknowledge the important role played by the students, teachers and administrators of the Middlesex, New Jersey School District.

Reference Note

1. Maher, Carolyn A. Assessing understanding of mathematics--a look at some alternatives. Paper presented at the annual meeting of National Council of Teachers of Mathematics, Cherry Hill, New Jersey, March, 1980.

References

- Blake, R. N. The effect of problem context upon the problem-solving processes used by field dependent and independent students: a clinical study. Dissertation Abstracts International. 1977, 37, 4191A-4192A.
- Brown, M. Multiway frequency tables--the log-linear model. In W. J. Dixon and M. B. Brown (Eds.), BMDP-79 Biomedical Computer Programs P-Series. Berkeley: University of California Press 1979.
- Bruner, Jerome S. The process of education. Cambridge: Harvard University Press, 1961.
- Haberman, S. Analysis of qualitative data, vol. 1, New York: Academic Press, 1978.
- Inhelder, B. and Piaget, J. The growth of logical thinking from childhood to adolescence. New York: Basic Books, 1958.
- Lester, Frank K. Research on mathematical problem solving. In Richard J. Shumway (Ed.), Research in Mathematics Education, Reston, Virginia: National Council of Teachers of Mathematics, Inc., 1980, 286-323.
- Maher, C. and O'Brien, J. A study of the relationship between long-term memory and formal cognitive development for eighth-grade students: a confirmation of Piagetian theory. Paper presented at the AERA Annual Meeting, Boston, April, 1980 (ERIC Document Reproduction Service No. ED 187 765).

Maher, C. and O'Brien, J. The association between Piagetian cognitive and long-term mnemonic levels and problem solving in mathematics. Paper presented at AERA Annual Meeting, Los Angeles, March, 1981 (ERIC Document Reproduction Service No. ED 202 917). (a)

Maher, C. and O'Brien, J. Piagetian operative level related to horizontality and its influence on short-term and long-term memory. A paper presented at the AERA Annual Meeting, Los Angeles, March, 1981 (ERIC Document Reproduction Service No. ED 202 570). (b)

Piaget, J. and Inhelder, B. Memory and intelligence. New York: Basic Books, 1973.

Piaget, J. and Inhelder, B., and Szeminska, A. The child's conception of geometry. New York: Basic Books, 1960.

Silver, Edward A. Student perceptions of relatedness among mathematical verbal problems. Journal of Research in Mathematics Education, 1979, 10, 195-210.

Upton, Graham, J. G. The analysis of cross-tabulated data. John Wiley & Sons, New York: 1978.

Witkin, Herman A., Oltman, Philip K., Raskin, Evelyn, and Karp, Stephen A. Manual. Palo Alto, California: Consulting Psychologists Press, 1971.

Table 1
Correlation Matrix: Yule's Q

	Proportional Logic Structure	Six-Month Recall of Combinatorial Logic Structure	Six-Month Recognition of Combinatorial Logic Structure	Cognitive Style Structure	Mathematical Problem Solving Level
Proportional Logic Structure	1.00	.88***	.43**	.51***	.62***
Six-Month Recall of Combinatorial Logic Structure		1.00	.__*	.59**	.65***
Six-Month Recognition of Combinatorial Logic Structure			1.00	.54***	.56***
Cognitive Style Structure				1.00	.74***
Mathematical Problem Solving Level					1.00

* χ^2 Statistic not computed due to an observed cell count of zero.

**p<.01

***p<.001

Table 2

Observed and Fitted Cell Counts for Cross-Classification for Proportional Logic Structure, Six-Month Memory Structures, Cognitive Style Structure and Mathematical Problem Solving Level

Proportional Logic Structure	Six-Month Memory Structures	Cognitive Style Structure	Mathematical Problem Solving Level	
			High ^a	Low ^b
Formal	Recall	Field Independent	17 (16.737)	1 (2.263)
		Field Dependent	4 (5.263)	4 (3.737)
	Recognition	Field Independent	15 (15.536)	5 (5.456)
		Field Dependent	2 (2.464)	4 (4.544)
	No Recognition	Field Independent	4 (5.037)	1 (.968)
		Field Dependent	7 (6.963)	6 (7.032)
Non Formal	Recall	Field Independent	2 (2.713)	1 (1.285)
		Field Dependent	0 (.287)	0 (.715)
	Recognition	Field Independent	12 (11.124)	2 (3.884)
		Field Dependent	8 (9.876)	19 (18.116)
	No Recognition	Field Independent	3 (4.823)	8 (7.175)
		Field Dependent	4 (4.823)	23 (24.825)

^a At or above grade level

^b Below grade level

Table 3

Ninety-Five Percent Confidence Intervals for Odds in Favor of Successful Problem Solving Level Conditioned on Proportional Logic Structure, Six-Month Memory Structures and Cognitive Style Structure

Proportional Logic Structure	Six-Month Memory Structures	Cognitive Style Structure	Odds in Favor of Successful Mathematical Problem Solving Level
Formal	Recall	Field Independent	7.39 +22.24 -5.55
		Field Dependent	1.41 +3.90 -1.03
	Recognition	Field Independent	2.84 +4.70 -1.77
		Field Dependent	.54 +2.01 -.43
	No Recognition	Field Independent	5.20 +40.60 - 4.61
		Field Dependent	.99 +1.83 -.64
Non Formal	Recall	Field Independent	2.11 +15.11 -1.85
		Field Dependent	.40 +30.11 -.39

Table 3 Continued

Proportional Logic Structure	Six-Month Memory Structures	Cognitive Style Structure	Odds in Favor of Suc- cessful Mathematical Problem Solving Level
	Recognition	Field Independent	2.86 $\begin{smallmatrix} +6.22 \\ -1.96 \end{smallmatrix}$
		Field Dependent	.54 $\begin{smallmatrix} +.64 \\ -.29 \end{smallmatrix}$
	No Recognition	Field Independent	.67 $\begin{smallmatrix} +1.46 \\ -.46 \end{smallmatrix}$
		Field Dependent	.12 $\begin{smallmatrix} +.28 \\ -.09 \end{smallmatrix}$

Table 4

47

Ninety-Five Percent Confidence Interval Estimates of the
Relative Odds of Successful Mathematical
Problem Solving Level

	Relative Odds for Successful Problem Solving Level	
Formal Proportional Logic Structure vs. Nonformal Proportional Logic Structure Given Recall	3.50	+28.10 -3.11
Formal Proportional Logic Structure vs. Nonformal Proportional Logic Structure Given Recognition	.99	+2.11 -.67
Formal Proportional Logic Structure vs. Nonformal Proportional Logic Structure Given No Recognition	7.74	+21.31 -5.67
Recall vs. Recognition Given Formal Proportional Logic Structure	2.59	+6.97 -1.89
Recall vs. Recognition Given Non Formal Proportional Logic Structure	.73	+5.30 -.64
Recognition vs. No Recognition Given Formal Proportional Logic Structure	.54	+1.83 -.42
Recognition vs. No Recognition Given Non Formal Proportional Logic Structure	4.26	+8.08 -2.79
Recall vs. No Recognition Given Formal Proportional Logic Structure	1.42	+4.42 -1.07
Recall vs. No Recognition Given Non Formal Proportional Logic Structure	3.14	+22.84 -2.76
Field Independent Cognitive Style Structure vs. Field Independent Cognitive Style Structure	5.25	+6.13 -2.83

Table 5
Correlation Matrix: Yule's Q

	MPS	GPS	PLS	CSS
Mathematical Problem Solving Level (MPS)	1.00	0.53**	0.40*	0.63***
Group Properties Structure (GPS)		1.00	0.01	0.36*
Proportional Logic Structure (PLS)			1.00	0.30
Cognitive Style Structure (CSS)				1.00

* $p < 0.05$

** $p < 0.01$

*** $p < 0.001$

Table 6

Observed and Fitted Cell Counts for the Cross Classification
for the Group Properties Structure, Proportional Logic
Structure, Cognitive Style Structure and
Mathematical Problem Solving Level

Group Properties, Structure	Propor- tional Logic Structure	Cognitive Style Structure	Mathematical Problem Solving Level	
			<u>Low</u> ^a	<u>High</u> ^b
Did not understand	Non-Formal	Field Dependent	22 (20.94)	5 (3.98)
		Field Independent	4 (7.12)	3 (5.36)
	Formal	Field Dependent	7 (8.91)	4 (3.69)
		Field Independent	5 (3.03)	4 (4.98)
Understood	Non-Formal	Field Dependent	13 (14.14)	7 (7.95)
		Field Independent	6 (4.81)	11 (10.72)
	Formal	Field Dependent	6 (6.02)	5 (7.38)
		Field Independent	0 (2.05)	11 (9.95)

^a Below grade level

^b At or above grade level

Table 7

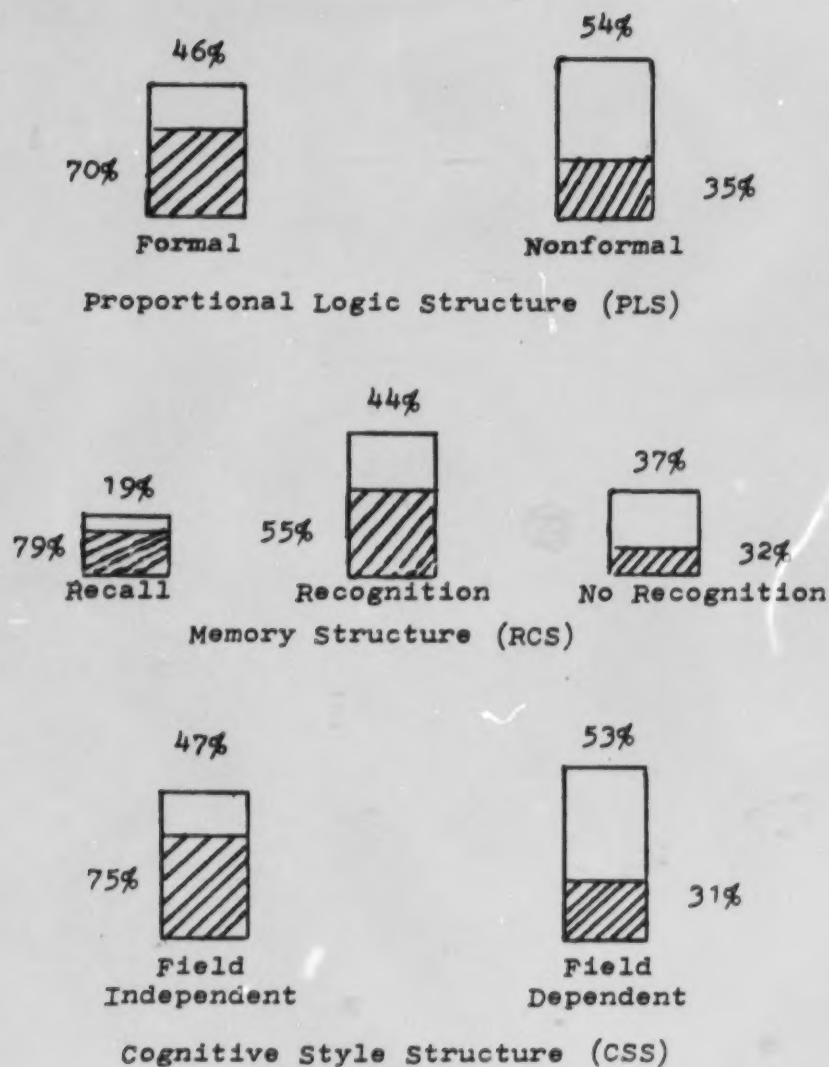
Ninety-Five Percent Confidence Intervals for Odds in Favor
 of Successful Mathematical Word Problem Solving
 Conditional on Group Properties Structure,
 Proportional Logic Structure and
 Cognitive Style Structure

Group Properties Structure	Proportional Logic Structure	Cognitive Style Structure	Odds in Favor of Successful Problem Solving
Did not understand	Non-Formal	Field Dependent	0.2 +0.4 -0.1
		Field Independent	0.8 +1.5 -0.5
	Formal	Field Dependent	0.4 +0.3 -0.3
		Field Independent	1.6 +5.2 -1.2
Understood	Non-Formal	Field Dependent	0.6 +0.8 -0.3
		Field Independent	2.2 +4.3 -1.5
	Formal	Field Dependent	1.2 +2.4 -0.8
		Field Independent	4.9 +5.6 -4.4

Table 8

Ninety Percent Confidence Interval Estimates of the Relative
Odds of Successful Mathematical Problem Solving

Subjects Being Compared	Relative Odds for Successful Problem-Solving Level
Group Properties Structure - Understood vs. Did Not Understand	3.0 +2.9 -1.5
Proportional Logic Structure - Formal vs. Non-Formal	2.2 +2.1 -1.1
Cognitive Style Structure - Field Independent vs. Field Dependent	4.0 +3.8 -2.0



% At or Above
Grade Level in
Problem Solving



% of Subjects with the
Labeled Assessment

Figure 1. Mathematical problem solving test performance (MPS) related to the following assessments: proportional logic structure, memory structure and cognitive style structure.

Proportion at or above grade level for mathematical
Problem Solving Test Performance (MPS) 53

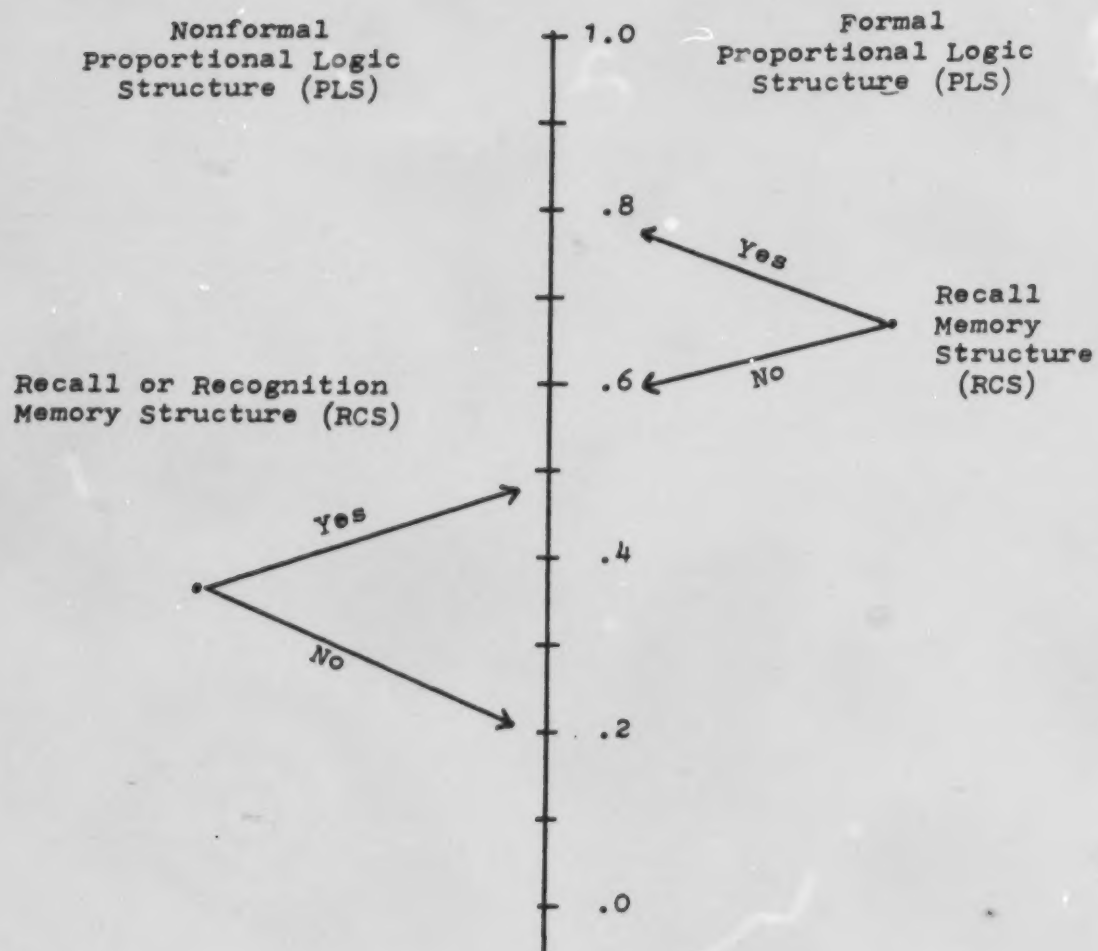


Figure 2. Mathematical problem solving achievement in terms of the following assessments: proportional logic structure and combinatorial logic structure.

Proportion at or above grade level for mathematical
Problem Solving Test Performance (MPS)

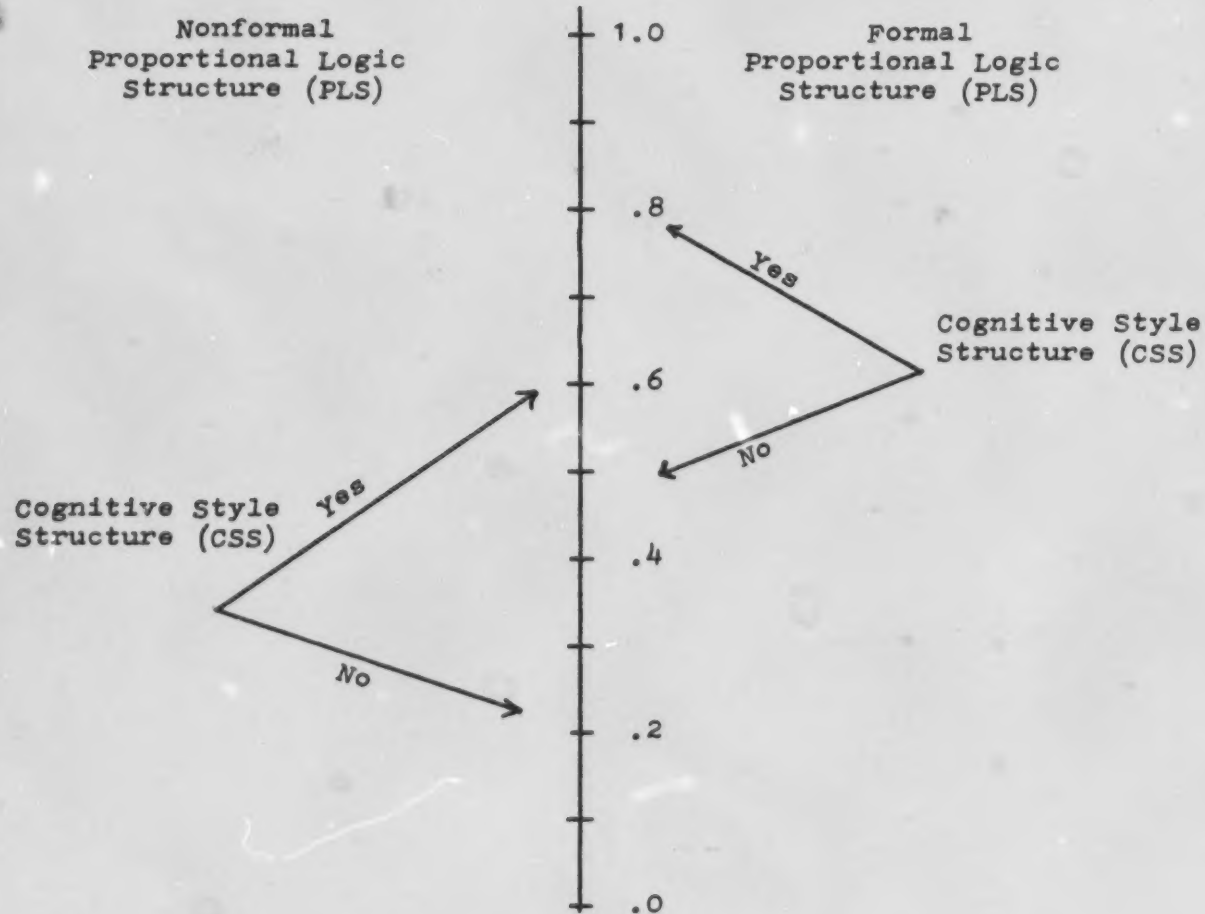


Figure 3. Mathematical problem solving achievement in terms of the following assessments: proportional logic structure and cognitive style structure.

Proportion at or above grade level for mathematical
problem solving test performance (MPS)

55

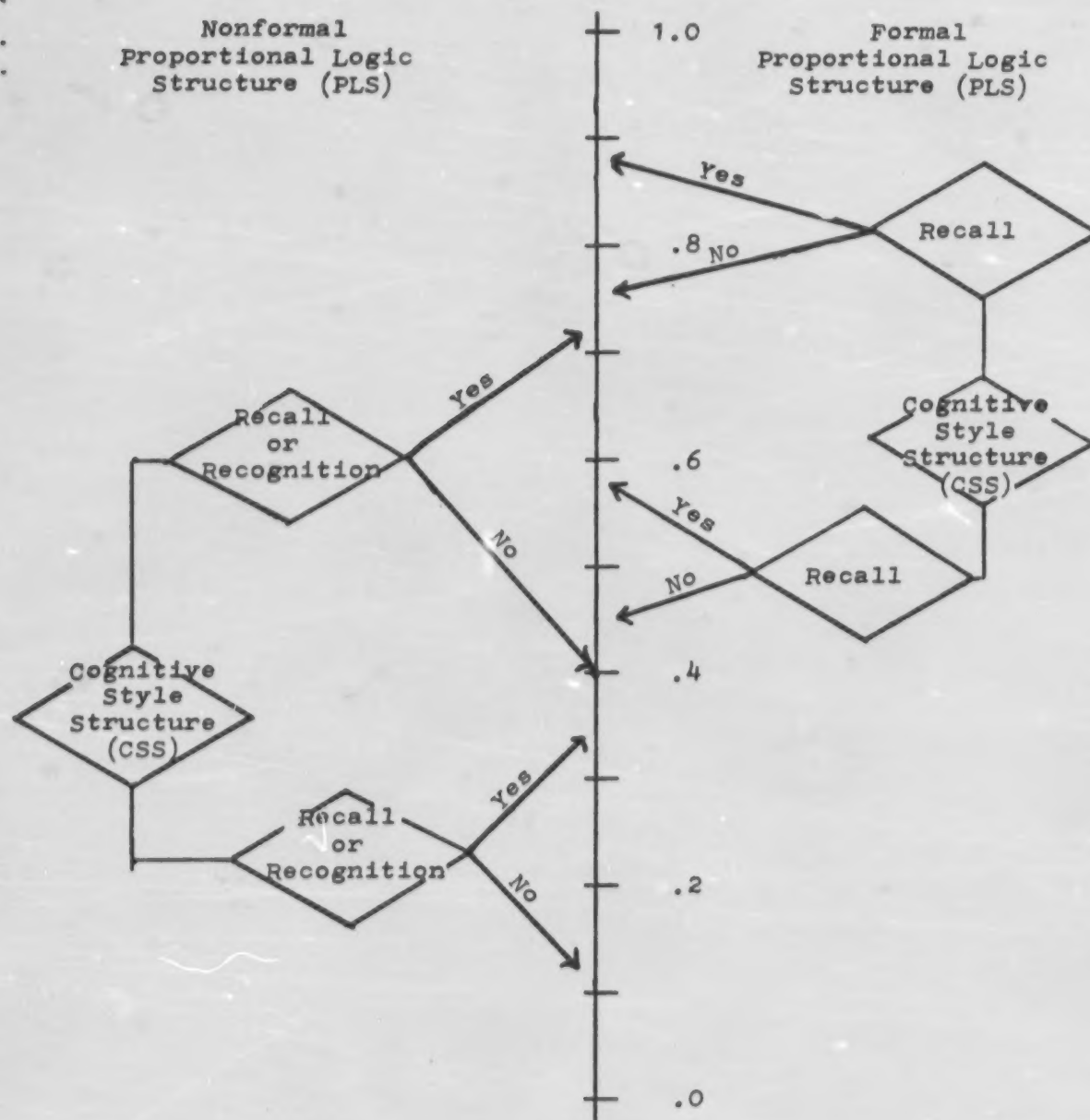


Figure 4. Mathematical problem solving achievement in terms of the following assessments: proportional logic structure, combinatorial logic structure and cognitive style structure.

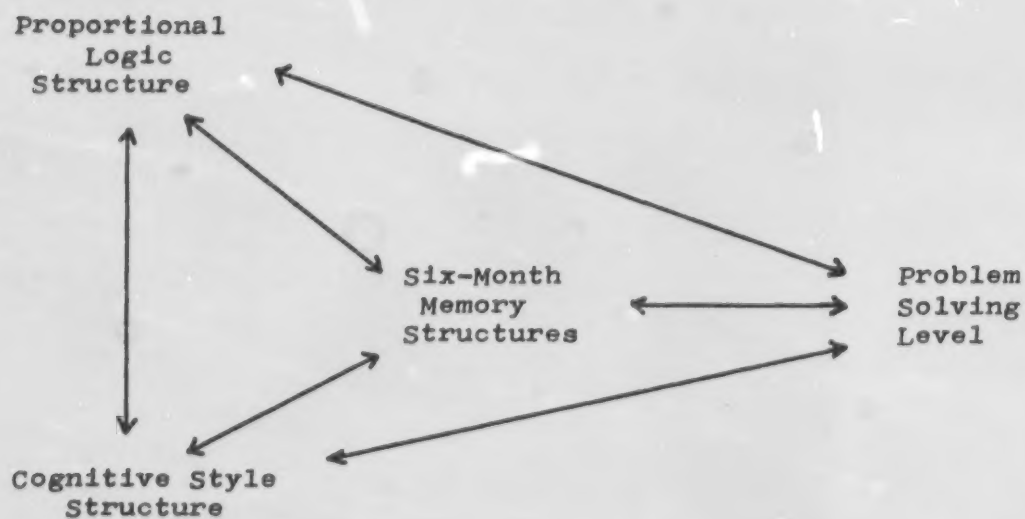


Figure 5. Relationships among proportional logic structure, six-month memory structures, cognitive style structure and mathematical problem solving level.

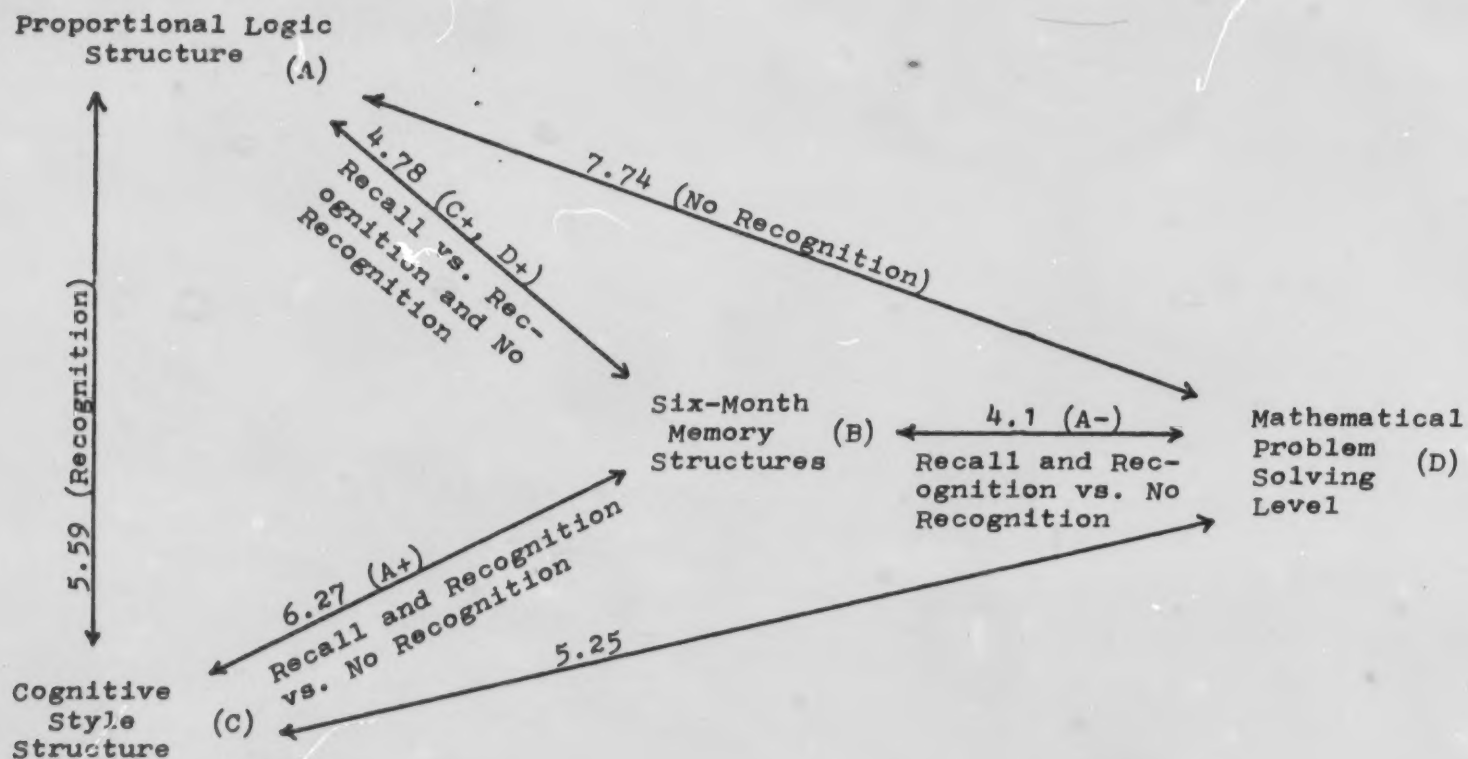
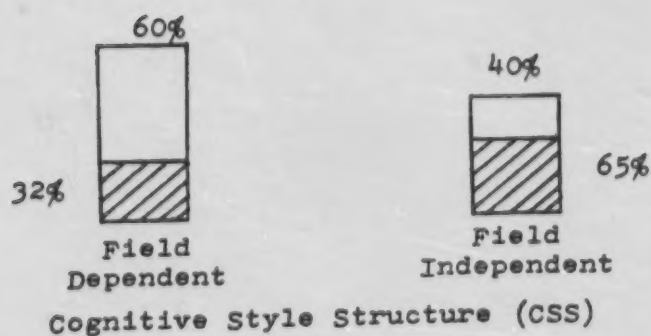
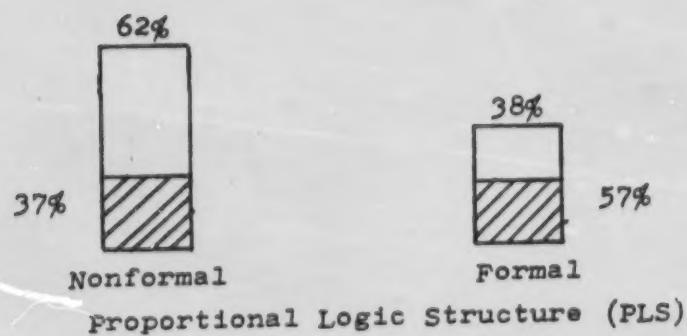
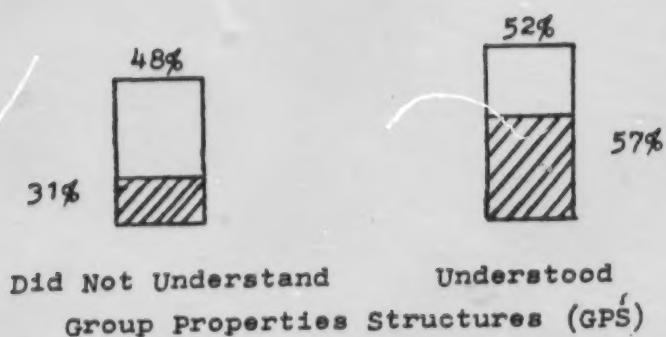


Figure 6. Path analysis illustrating the association among proportional logic structure, six-month memory structures, cognitive style structure and mathematical problem solving level.



% At or Above
Grade Level in
Problem Solving



% of Subjects with the
Labeled Assessment

Figure 7. Mathematical problem solving (MPS) test performance related to the following assessments: group properties structure, proportional logic structure and cognitive style structure.

Proportion at or above grade level for mathematical
problem solving test performance (MPS)

Did Not Understand
Group Properties
Structures
(GPS)

Understood
Group Properties
Structures
(GPS)

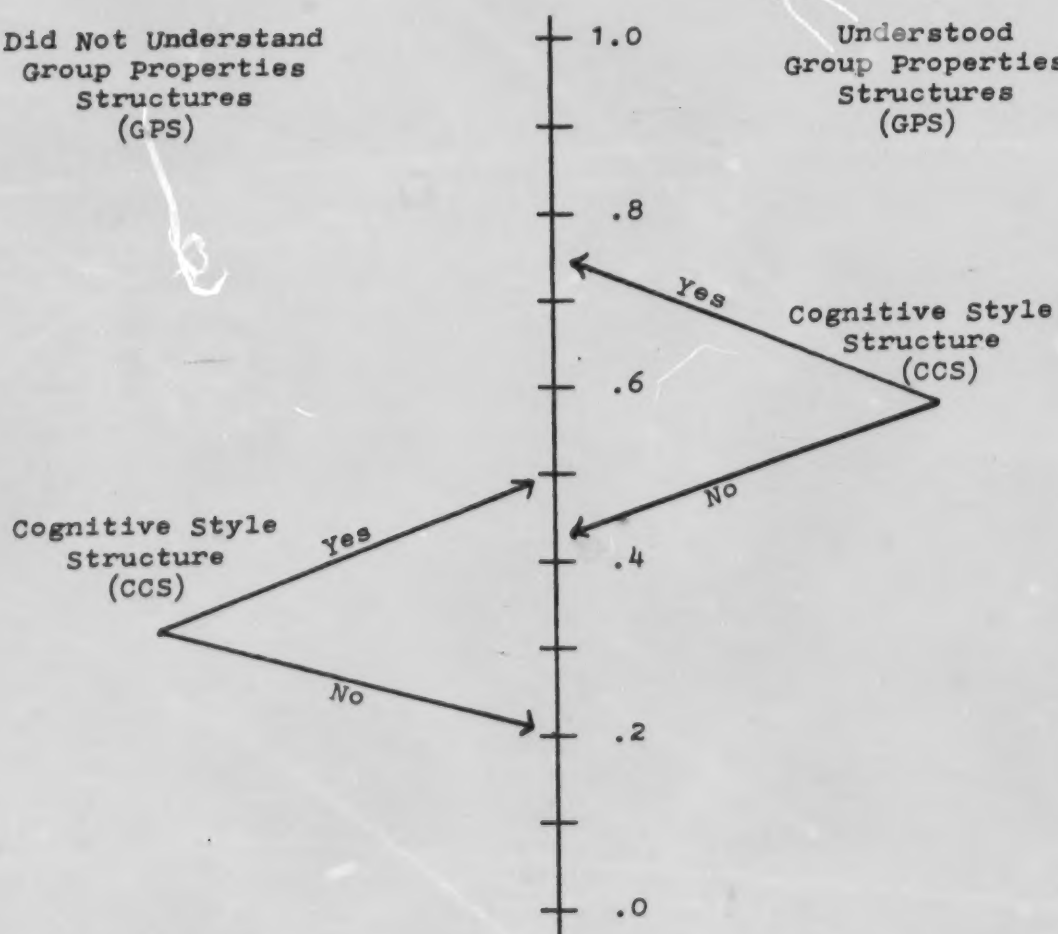


Figure 8. Mathematical problem solving achievement in terms of the following assessments: group properties structures and cognitive style structure.

Proportion at or above grade level for mathematical
problem solving test performance (MPS)

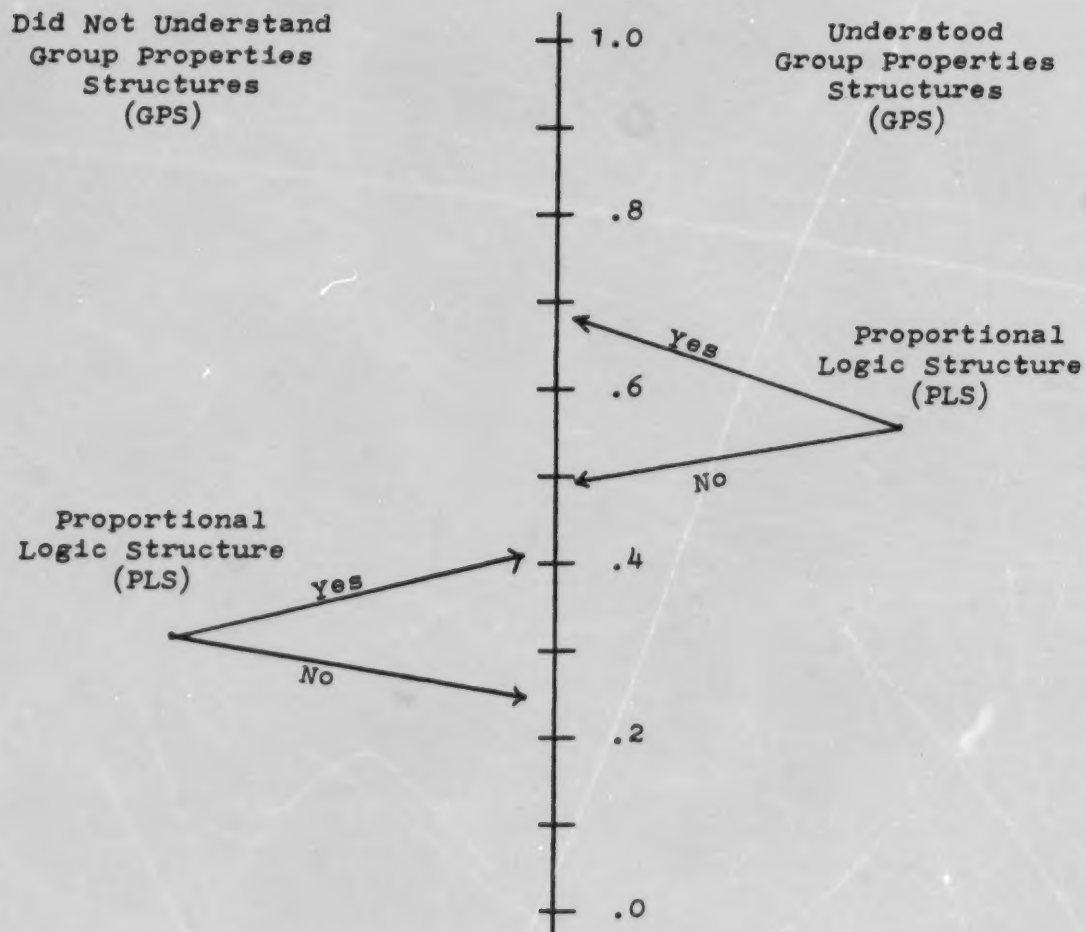


Figure 9. Mathematical problem solving achievement in terms of the following assessments: group properties structure and proportional logic structure.

Proportion at or above grade level for mathematical
problem solving test performance (MPS)

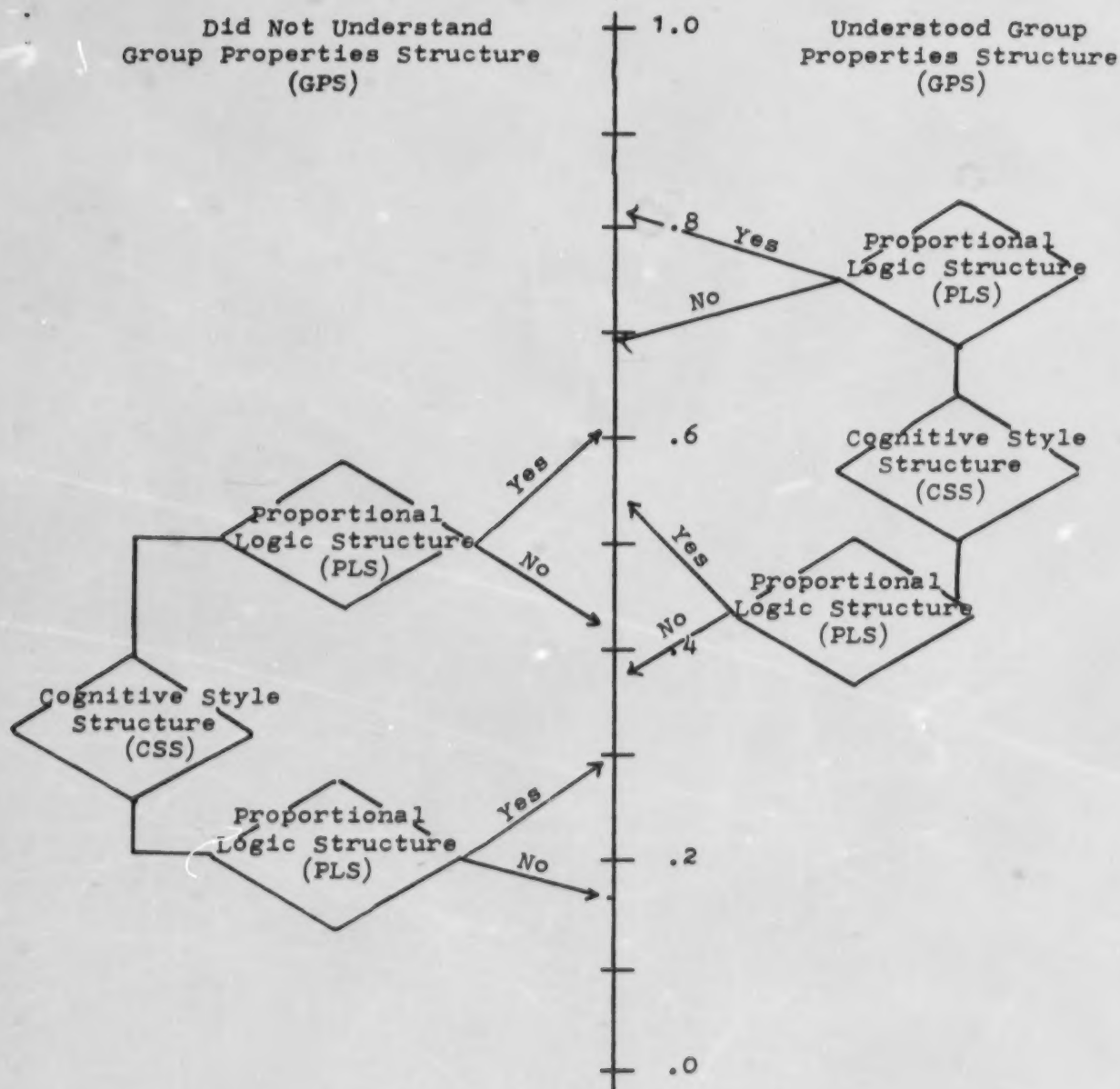


Figure 10. Mathematical problem solving achievement in terms of the following assessments: group properties structure, proportional logic structure and cognitive style structure.

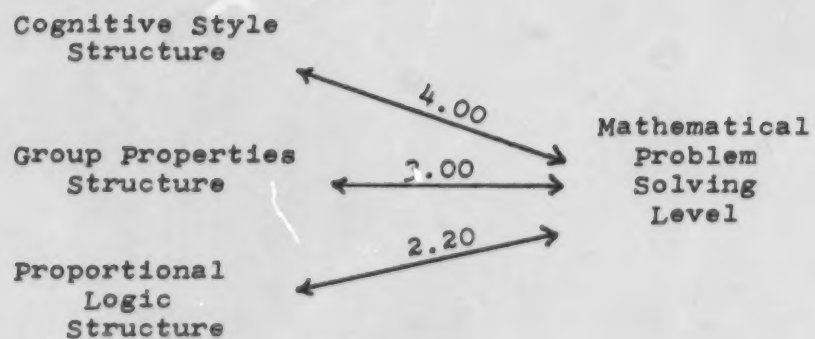


Figure 11. Path analysis illustrating the association among cognitive style structure, group properties structure, proportional logic structure and mathematical problem solving level.

APPENDIX A: PROTOCOL FOR BALANCE BEAM TASK

Name _____
 Clinician _____
 Date _____
 Sex M F
 Age _____

	Clinician	Subject	Placement	Observed Method	Best Explanation
1	4 wt. @ 5	B. 4 wt.	4 4 5 □□□□□	Haphazard T&E Thoughtful	Eq. weights/Eq. dists. None Additive Multiplication Proportional Other _____
2	Remove 2 wt. from a 4 wt. Ask subject if scale will still balance. If not, in which direc- tion will it tip?	Predicts tip	Correct Incorrect		None More weight Heavy closer/light farther Other _____
		B. 2 wt.	4 2 5 □□□□□	Haphazard T&E Thoughtful	None Additive Multiplication Proportional Halves Twice Other _____

	Clinician	Subject	Placement	Observed Method	Best Explanation
3	Add 2 wt. to the 4 wt. Ask subject if scale will still balance. If not, in which direction will it tip?	Predicts tip	Correct Incorrect		None More weight Heavy closer/light farther Other _____
	6 wt. @ 1	B. 2 wt.	<u>6 2</u> 1 □□□□□	Haphazard T&E Thoughtful	None Additive Multiplication Proportional Thirds Thrice Other _____
4	4 wt. @ 9	B. 6 wt.	<u>4 6</u> 9 □□□□□	Haphazard T&E Thoughtful	None Additive Multiplication Proportion Other _____

	Clinician	Subject	Placement	Observed Method	Best Explanation
5	4 wt. @ 4	B. 8 wt.	$\begin{array}{r} 4 \quad 8 \\ \hline 4 \quad \square\square\square\square\square \end{array}$	Haphazard T&E Thoughtful	None Additive Multiplication Proportion Halves Twice Other _____ _____
	TRAINING "Notice that the 8 wt. is two times the 4 wt. What do you notice about the distances?"				4 is more than 2 2 + 2 = 4 2 is half of 4 4 is twice 2 Nothing

	Clinician	Subject	Placement	Observed Method	Best Explanation
6	3 wt. @ 6	B. 9 wt.	$\begin{array}{r} 3 \ 9 \\ \hline 6 \end{array}$ □□□□□	Haphazard T&E Thoughtful	None Additive Multiplication Proportion Thirds Thrice Other _____ _____
	TRAINING "Notice that the 9 wt. is three times the 3 wt. What do you notice about the distances?"				6 is more than 2 $2 + 4 = 6$ 6 is three times 2 2 is one-third of 6 Nothing

	Clinician	Subject	Placement	Observed Method	Best Explanation
7	5 wt. @ 4	B. 2 wt.	<u>5 2</u> 4 □□□□□	Haphazard T&E Thoughtful	None Additive Multiplication Proportional Other _____
	TRAINING "Notice that 5 is 2-1/2 times the 2 wt. What do you notice about the distances?"	-----	-----	-----	10 is more than 4 4 + 6 = 10 10 is 2-1/2 times 4 4 is 2/5 of 10 Nothing
8	3 wt. @ 8	B. 6 wt.	<u>3 6</u> 8 □□□□□	Haphazard T&E Thoughtful	None Additive Multiplication Proportional Halves Twice Other _____
9	4 wt. @ 3	B. 6 wt.	<u>4 6</u> 3 □□□□□	Haphazard T&E Thoughtful	None Additive Multiplication Proportional Other _____

	Clinician	Subject	Placement	Observed Method	Best Explanation
10	6 wt. @ 3	B. 2 wt.	$\begin{array}{r} 6 \ 2 \\ \hline 3 \ \square\square\square\square\square \end{array}$	Haphazard T & E Thoughtful	None Additive Multiplication Proportional Thirds Thrice Other _____ _____
11	4 wt. @ 6	B. 3 wt.	$\begin{array}{r} 4 \ 3 \\ \hline 6 \ \square\square\square\square\square \end{array}$	Haphazard T & E Thoughtful	None Additive Multiplication Proportional Other _____ _____
12	4 wt. @ 4	B. 2 wt.	$\begin{array}{r} 4 \ 2 \\ \hline 4 \ \square\square\square\square\square \end{array}$	Haphazard T & E Thoughtful	None Additive Multiplication Proportional Halves Twice Other _____ _____

	Clinician	Subject	Placement	Observed Method	Best Explanation
13	3 wt. @ 3	B. 9 wt.	$\frac{3}{3} \frac{9}{\square\square\square\square\square}$	Haphazard T&E Thoughtful	None Additive Multiplication Proportional Thirds Thrice Other _____ _____
14	4 wt. @ 7	B. 2 wt.	$\frac{4}{7} \frac{2}{\square\square\square\square\square}$	Haphazard T&E Thoughtful	None Additive Multiplication Proportional Halves Twice Other _____ _____
15	"How does the balance beam work?"		Comment:		None Additive Multiplication Heavy closer/light farther Proportional Other _____ _____

16 Clinician - Record general impression of child in relation to this task.

Balance Beam Ratings

<u>TIP</u>	<u>2:1</u>	<u>3:1</u>	<u>non-int.</u>	<u>Explain</u>	<u>Rating</u>
3) yes	2) p n	3) p n	4) p n	2:1	
no	5) p n	6) p n	7) p n	3:1	
yes	8) p n	10) p n	9) p n	non-int.	
no	<u>12) p n</u>	13) p n	11) p n	not prop.	
	14) p n				

Name - _____

3) yes	2) p n	3) p n	4) p n	2:1	
no	5) p n	6) p n	7) p n	3:1	
yes	8) p n	10) p n	9) p n	non-int.	
no	<u>12) p n</u>	13) p n	11) p n	not prop.	
	14) p n				

Name - _____

3) yes	2) p n	3) p n	4) p n	2:1	
no	5) p n	6) p n	7) p n	3:1	
yes	8) p n	10) p n	9) p n	non-int.	
no	<u>12) p n</u>	13) p n	11) p n	not prop.	
	14) p n				

Name - _____

APPENDIX B: PROTOCOL FOR THE GROUP
PROPERTIES TASK

GUIDE FOR THE FIGURE ROTATION TASK

EQUIPMENT: Cards and 2 sets of dolls

I. Demonstration of the use of the cards

"Facing front" means boy and girl are facing you.

"(Will you put the dolls in the face front position?)"

Either boy moves about face, or girl moves about face, or boy and girl move about face, or neither move at all, according to an instruction given on a command card.

Here are the command cards. (Show student cards.)

Only Boy Turns

Only Girl Turns

Both Turn

Nobody Turns

Let me show you how a card works. Select and demonstrate.

Only Girl Turns

The command card instructs you to turn the girl about face. Notice the boy does not move.

Let's try another card. But before we do so, both dolls must be in the facing front position.

Have the student demonstrate.

Only Boy Turns

Make clear that dolls are always returned to the facing front position before each new command card is demonstrated.

II. Verification of understanding of the correspondence between a movement and a card

Demonstrate the position "both turn" with dolls.

A. Ask student to place dolls in facing front position.

B. You turn both dolls about face.

D. Continue this until all possible command cards have been matched by the student.

(For "nobody turns" be sure both dolls are clearly in front of student and student is aware that a command has been given.)

III. Composition

Let's see what happens if the rules are changed. Suppose we use two command cards in succession, a first followed by a second. Before the first command, the dolls must be in the facing front position. However, before the second command, it is not permitted to return the dolls to the facing front position.

IV. Verification of understanding composition

Here's an example. Choose then . You demonstrate with the dolls for the student. Then Ask: "Choose one card that would have produced the same result by itself from a facing front position."

If the child cannot correctly select the card, leave the demonstration dolls with position unchanged. Beginning with the other set of dolls in the facing front position, ask the student to select the command card that would produce the result you've just obtained within the first set of dolls.

Repeat the second example using and as first and second commands respectively.

V. Using the table

Along the left side are the first commands; across the top are the second. Before the first command the dolls are in the facing front position.

After the first command (OBT) and second command (OBT), the dolls will be in a certain position. Into the box (point and show box) place the single command card which starting from a facing front position would have produced that same position by itself.

After the first command (OBT) and the second command (OGT) the dolls will be in the certain position. Into the box (point and show box) place the single command card which, starting from the facing front position, would have produced that same position, by itself.

After the first command (OBT) and the second command (BT) the dolls will be in a certain position. Into the box (point and show box) place the single command card which, starting from the facing front position, would have produced that same position by itself.

NOTE: Tell students to feel free to use completed table or dolls at any time to answer any questions.

PROTOCOL FOR THE FIGURE ROTATION TASK

Student's Name _____

Date _____

Sex _____

Clinician _____

Record Student responses:

Task 1

Circle appropriate method used
by students:

Followed by	OBT	OGT	BT	NT
OBT				
OGT				
BT				
NT				

Mostly/total manipulative

Mostly/total mental

Mixed manipulative/mental

Record first correct or incorrect response of student
in the table on ditto. (i.e., OBT, OGT, BT, NT).

*for each incorrect answer, probe until correct.

Task 2

COMMUTATIVE

I

- A) "Suppose we reverse the order of the commands so that the first command is now the second and the second is the first. What will happen if we reverse the order of the commands?"

_____ successful _____ unsuccessful*

*if unsuccessful, go to II

B) "Can you give me an example?"

$$\square \circ \square = \square \circ \square$$

_____ successful _____ unsuccessful*

*if unsuccessful, go to II

C) "Can you think of any other examples?"

$$\square \circ \square = \square \circ \square \quad \square \circ \square = \square \circ \square$$

$$\square \circ \square = \square \circ \square$$

_____ successful _____ unsuccessful*

*if unsuccessful, go to II

D) "Can you find an example where reversing the order of the commands makes a difference in the answer?"

_____ successful _____ unsuccessful*

Explain _____

$$\square \circ \square = \square \circ \square$$

*if unsuccessful, go to II

E) "Suppose we reverse the order of the commands so that the first command is now the second and the second is the first. What will happen if we reverse the order of the commands?"

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, go to II

II

- A) "Let's explore a first command followed by a second command." (Clarify IA again)

$$\boxed{\text{OGT}} \circ \boxed{\text{OBT}} = \boxed{}$$

"Can you reverse the order of these commands?"

$$\boxed{} \circ \boxed{} = \boxed{} = \boxed{}$$

"What happens when we reverse the order of these commands?"

Explain _____

*if unsuccessful, go to III

- B) "Can you think of any other examples?"

$$\boxed{} \circ \boxed{} = \boxed{} \circ \boxed{}$$

$$\boxed{} \circ \boxed{} = \boxed{} \circ \boxed{}$$

$$\boxed{} \circ \boxed{} = \boxed{} \circ \boxed{}$$

*if unsuccessful, go to III

- C) "Can you find an example where reversing the order of the commands makes a difference in the answer?"

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, go to III

- D) "Suppose we reverse the order of the commands so that the first command is now the second and the second

is the first. What will happen if we reverse the order of the commands?"

_____ successful _____ unsuccessful*
 *if unsuccessful, go to III

III

A) Provide other examples:

o = o

o = o

o = o

DO NOT WORK THEM OUT.

B) Let's explore a first command followed by a second command." (Explain)

"Can you reverse the order of these commands?"

"What happens when we reverse the order of these commands?"

Explain _____

*if unsuccessful, go to IV

C) "Can you find an example when reversing the order of the commands makes a difference in the answer?"

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, go to IV

Explain _____

*if unsuccessful, go to IV

- D) "Suppose we reverse the order of the commands so that the first command is now the second and the second is the first. What will happen if we reverse the order of the commands?"

_____successful _____unsuccessful*

*if unsuccessful, go to IV

IV

- A) Work out examples in part IIIA
- B) "Let's explore a first command followed by a second command." (Explain) "Can you reverse the order of these commands?" "What happens when we reverse the order of these commands?"

Explain _____

- C) "Can you find an example when reversing the order of the commands makes a difference in the answer?"

_____successful _____unsuccessful

Explain _____

- D) "Suppose we reverse the order of the commands so that the first command is now the second and the second is the first. What will happen if we reverse the order of the commands?"

_____ successful _____ unsuccessful

Task 3

IDENTITYI

"So far we have considered a first command followed by a second command. Continue to think of one command followed by a second command."

- A) "I'm looking for a single command that is special in a certain way. When the single, special command is used with any other command, it leaves the position arrived at by the original command unchanged." (Repeat statement if necessary.)

"Do you have any idea what the special command would be?" ☐

_____ successful _____ unsuccessful*

*if unsuccessful, go to II

- B) "In this case ☐ NT is the special command. In order for this command to be really special, it has to work for all commands."

"Do you think the command ☐ NT that you have chosen leaves the position arrived at by every command unchanged?"

_____ successful _____ unsuccessful*

Explain _____

OR
EXAMPLES

☐ o ☐ = ☐

☐ o ☐ = ☐

☐ o ☐ = ☐

☐ o ☐ = ☐

*if unsuccessful, go to II

- C) "What happens when ☐ NT is the first command followed by a second command? Does ☐ NT still behave as the special command?"

Explain _____

OR
EXAMPLES

*if unsuccessful, go to II

II

- A) DEMONSTRATE: ☐ BT o ☐ = ☐ BT

"What command would follow the ☐ BT command such that the resulting position will be ☐ BT ?"

_____ successful _____ unsuccessful*

*if unsuccessful, go to III

- B) "In this case ☐ NT is the special command. In order for this command to be really special, it has to work for all commands."

"Do you think the comand ☐ NT that you have chosen leaves the position arrived at by every command unchanged?"

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, probe with examples

$$\boxed{} \circ \boxed{} = \boxed{}$$

$$\boxed{} \circ \boxed{} = \boxed{}$$

$$\boxed{} \circ \boxed{} = \boxed{}$$

$$\boxed{} \circ \boxed{} = \boxed{}$$

*if unsuccessful, go to III

- C) "What happens when **NT** is the first command followed by a second command? Does **NT** still behave as the special command?"

Explain _____

OR
EXAMPLES

$$\boxed{\text{NT}} \circ \boxed{} = \boxed{}$$

$$\boxed{\text{NT}} \circ \boxed{} = \boxed{}$$

$$\boxed{\text{NT}} \circ \boxed{} = \boxed{}$$

$$\boxed{\text{NT}} \circ \boxed{} = \boxed{}$$

*if unsuccessful, probe with examples (Record student examples.)

*if unsuccessful, go to III

III

- A) Provide other examples for student but let student attempt to work them out.

$$\boxed{\text{DBT}} \circ \boxed{} = \boxed{\text{DBT}}$$

$$\boxed{\text{OGT}} \circ \boxed{} = \boxed{\text{OGT}}$$

$$\boxed{\text{NT}} \circ \boxed{} = \boxed{\text{NT}}$$

$$\boxed{\text{BT}} \circ \boxed{} = \boxed{\text{BT}}$$

- B) "In this case **NT** is the special command. In order for this command to be really special, it has to work for all commands. Do you think the command **NT** that you have chosen leaves the position arrived at by every command unchanged?"

_____ successful _____ unsuccessful*

OR
EXAMPLES

☐ o ☐ = ☐ o ☐

☐ o ☐ = ☐ o ☐

☐ o ☐ = ☐ o ☐

*if unsuccessful, probe with examples

- C) "What happens when **NT** is the first command followed by a second command? Does **NT** still behave as the special command?"

Explain _____

OR
EXAMPLES

*if unsuccessful, probe with examples

*if unsuccessful, to go IV

IV

- A) Work out examples in IIIA.

OBT o ☐ = **OBT**

OGT o ☐ = **OGT**

NT o ☐ = **NT**

BT o ☐ = **BT**

- B) "In this case **NT** is the special command. In order for this command to be really special, it has to work for all commands. Do you think the command **NT** that you have chosen leaves the position arrived at by every command unchanged?"

_____ successful _____ unsuccessful

Explain _____

OR
EXAMPLES

☐ o ☐ = ☐ o ☐

☐ o ☐ = ☐ o ☐

- C) "What happens when **NT** is the first command followed by a second command? Does **NT** still behave as the special command?"

Explain _____

OR
EXAMPLES

☐ o ☐ = ☐ o ☐

☐ o ☐ = ☐ o ☐

Task 4

INVERSE

I

- A) "I'm looking to pair each command with a partner

command so that the result of the command and its partner command will be the same as if only the special command **NT** had been given."

"Can you pair each command with a partner command so that the result will be the special command **NT**?"

_____ successful _____ unsuccessful*

$$\boxed{\text{OBT}} \circ \boxed{} = \boxed{\text{NT}}$$

$$\boxed{\text{OGT}} \circ \boxed{} = \boxed{\text{NT}}$$

$$\boxed{\text{BT}} \circ \boxed{} = \boxed{\text{NT}}$$

$$\boxed{\text{NT}} \circ \boxed{} = \boxed{\text{NT}}$$

*if unsuccessful, go to II

- B) "Is it possible for a command to have more than one partner command so that the result will be the special command **NT**?"

_____ successful _____ unsuccessful*

*if unsuccessful, go to II

- C) "If all examples in IA are not done by the student ask: Can you find a command that does not have a partner command?"

Explain _____

(if unsuccessful, go to II)

II

- A) "Have you any idea what command would follow **OBT** so that the result will be the special command **NT**?"

$$\boxed{\text{OBT}} \circ \boxed{} = \boxed{\text{NT}}$$

_____ successful _____ unsuccessful*

*if unsuccessful, go to III

☐ OGT o ☐ = ☐ NT

☐ BT o ☐ = ☐ NT

☐ NT o ☐ = ☐ NT

_____ successful _____ unsuccessful*

*if unsuccessful, pick another command card and say:

- B) "Have you any idea what command would follow ☐ so that the result will be the special command ☐ NT ?"

_____ successful _____ unsuccessful*

*if unsuccessful, go to III

- C) "Is it possible for a command to have more than one partner command so that the result will be the special command ☐ NT ?"

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, go to III

- D) "I'm looking to pair each command with a partner command so that the result of the command and its partner command will be the same as if only the special command ☐ NT had been given."
- "Can you pair each remaining command with a partner command so that the result will be the special command ☐ NT ?"

_____ successful _____ unsuccessful*

*if unsuccessful, go to III

III

- A) "Notice that if the command **OBT** is followed by the command **OBT** the result will be the special command **NT**. **OBT** is the partner command for **OBT**. Can you find a partner command for **OBT** such that when **OBT** is followed by its partner command the result will be the special command **NT** ?"

_____ successful _____ unsuccessful*

$$\boxed{\text{OBT}} \circ \boxed{} = \boxed{\text{NT}}$$

*if unsuccessful, go to IV

- B) Continue asking for partners for the remaining commands.

$$\boxed{\text{BT}} \circ \boxed{} = \boxed{\text{NT}}$$

$$\boxed{\text{NT}} \circ \boxed{} = \boxed{\text{NT}}$$

_____ successful _____ unsuccessful*

*if unsuccessful, go to IV

- C) "Is it possible for a command to have more than one partner command so that the result will be the special command **NT** ?"

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, go to IV

- D) "Can you pair each command with a partner command so that the result will be the special command **NT**?"

_____successful _____unsuccessful*

*if unsuccessful, go to IV

IV

- A) "Notice that if the command **OGT** is followed by the command **OGT** the result will be the special command **NT**.

OGT is the partner command for **OGT**. What is the partner

BT? **BT** o = **NT** for **NT**? **NT** o = **NT**

- B) "Is it possible for a command to have more than one partner command so that the result will be the special command **NT**?"

_____successful _____unsuccessful

Explain _____

- C) "Can you pair each command with a partner command so that the result will be the special command **NT**?"

_____successful _____unsuccessful

Task 5

ASSOCIATIVITY

Introduction

"So far we have talked about a first command followed by a second command. Let's explore what happens when we work with three commands."

- A) "Can you give me an example of a first command followed by a second command followed by a third command?"

_____ successful _____ unsuccessful*

o o

*if unsuccessful, select 3 cards for student

- B) Using the student's or your example task, ask the student how he/she would work out the example.

"Using the term 'grouping' point out to the student the process he/she is using."

o o

o

_____ successful _____ unsuccessful*

(Any attempt by the student to change the order of the commands must be discouraged. Student should be instructed that reversing the order is against the rules.)

RECORD if student attempts to change the order:

*if unsuccessful, illustrate grouping for the student and if necessary return to the use of dolls

- C) "Using the same example in (B) ask the student if he/she can group the commands in a different way to solve the problem without removing the previous work. Remind student that changing the order is against the rules."

o o

o

_____ successful _____ unsuccessful*

RECORD if student attempts to change the order:

*if unsuccessful, demonstrate the different way to grouping and if necessary return to the use of dolls

I

- A) "What will happen if we change the grouping of the three commands?"

_____successful _____unsuccessful*

Explain _____

*if unsuccessful, go to II

- B) "Can you give an example?"

$$\begin{array}{ccccccc} \boxed{} & \circ & \boxed{} & \circ & \boxed{} & = & \boxed{} & \circ & \boxed{} & \circ & \boxed{} \\ & & \boxed{} & \circ & \boxed{} & & \boxed{} & \circ & \boxed{} & & \\ & & \boxed{} & & & & \boxed{} & & & & \end{array}$$

*if unsuccessful, go to II

- C) Ask students to give another example: Record example.

*if unsuccessful, go to II

- D) "What will happen if we change the grouping of the three commands?"

_____successful _____unsuccessful

Explain _____

*if unsuccessful, go to II

II

- A) "Let's explore changing the grouping of three commands, without changing the order."

() o

- B) "Can you change the grouping of these commands, without changing the order?"

o o

_____successful _____unsuccessful*

*if unsuccessful, go to III

- C) "Can you give another example of grouping three commands in two different ways without changing the order of the commands?"

o o	=	o o
o		o

*if unsuccessful, go to III

III

- A) Provide examples for the student, but do not work them out.

OBT o OGT o BT	=	OBT o OGT o BT
o		o

_____successful _____unsuccessful*

RECORD if student changed order:

*if unsuccessful, go to IV

- B) "Let's do another example." If student cannot provide an example, provide one.

$$\begin{array}{ccccc} \boxed{} \circ \boxed{} \circ \boxed{} & = & \boxed{} \circ \boxed{} \circ \boxed{} \\ \boxed{} \circ \boxed{} & & \boxed{} \circ \boxed{} \\ \boxed{} & & \boxed{} \end{array}$$

_____ successful _____ unsuccessful*

RECORD if student changed order:

*if unsuccessful, go to IV

- C) "Do you think that anything will happen if we change the grouping of the three commands?"

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, go to IV

IV

- A) Work out example in IIIA with the student.

$$\begin{array}{ccccc} (\boxed{\text{OBT}} \circ \boxed{\text{OGT}}) \circ \boxed{\text{BT}} & = & \boxed{\text{OBT}} \circ (\boxed{\text{OGT}} \boxed{\text{BT}}) \\ \boxed{\text{BT}} \circ \boxed{\text{BT}} & & \boxed{\text{OBT}} \circ \boxed{\text{OBT}} \\ \boxed{\text{NT}} & & \boxed{\text{NT}} \end{array}$$

- B) Have student work out or help student work out another example.
- C) "Do you think that anything will happen if we change the grouping of the three commands?"

_____successful _____unsuccessful

Explain _____

Task 6

CLOSURE

I

- A) "Let's review, what were the original commands?"

_____successful _____unsuccessful*

☐ ☐ ☐ ☐

*if unsuccessful, refer to table and ask what commands we started with before filling in the table.

☐ ☐ ☐ ☐

(Keep these cards out and refer to them when asking questions.)

- B) "If one command is followed by another command, will the position that results always be one that could have been reached by one of the original commands by itself?" (Repeat question, if necessary.)

_____successful _____unsuccessful*

Explain _____

OR
RECORD EXAMPLES

$$\boxed{} \circ \boxed{} = \boxed{} \qquad \boxed{} \circ \boxed{} = \boxed{}$$

$$\boxed{} \circ \boxed{} = \boxed{}$$

*if unsuccessful, go to II

- C) "Can you find one command followed by another command such that the resulting position could not have been reached by one of the original commands by itself?"

_____ successful _____ unsuccessful*

Explain _____

OR
RECORD EXAMPLES

$$\boxed{} \circ \boxed{} = \boxed{} \qquad \boxed{} \circ \boxed{} = \boxed{}$$

$$\boxed{} \circ \boxed{} = \boxed{}$$

*if unsuccessful, go to II

- D) "If one command is followed by another command, will the position that results always be one that could have been reached by one of the original commands by itself?"

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, go to II

II

A) Let student work these examples

$$\boxed{\text{OBT}} \circ \boxed{\text{OGT}} = \boxed{}$$

$$\boxed{\text{BT}} \circ \boxed{\text{OBT}} = \boxed{}$$

"Could the resulting position of the above examples have been reached by one of the original commands by itself?"

(Emphasize we are talking about original commands.)

_____ successful _____ unsuccessful*

Explain _____

*if unsuccessful, go to III

B) "If one command is followed by another command, will the position that results always be one that could have been reached by one of the original commands by itself?"

_____ successful _____ unsuccessful*

Explain _____

OR
RECORD STUDENT'S
EXAMPLES

*if unsuccessful, to to III

III

A)

$$\boxed{\text{OBT}} \circ \boxed{\text{OGT}} = \boxed{\text{BT}}$$

"Notice that the command OBT o OGT results in the command BT. BT is one of the original commands."

- B) "If one command is followed by another command, will the position that results always be one that could have been reached by one of the original commands by itself?"

_____successful _____unsuccessful

Explain _____

OR
RECORD STUDENT'S
EXAMPLES

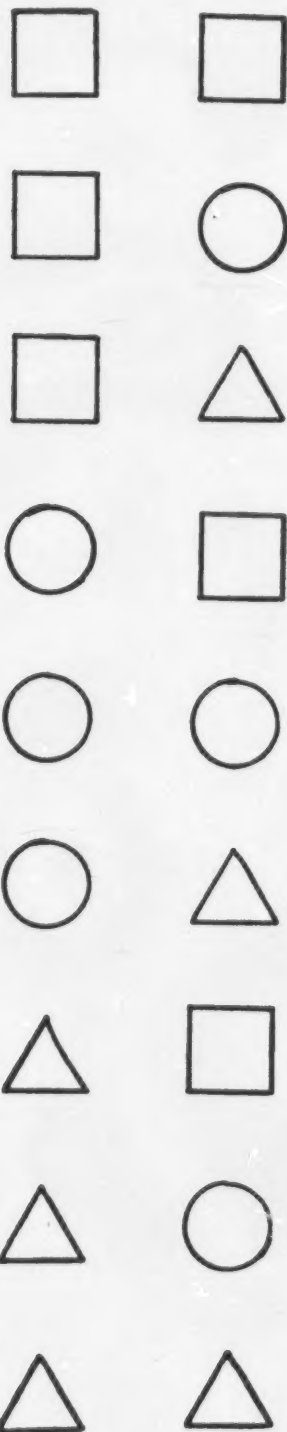
IV

IF THE CHILD HAS NOT SUCCEEDED IN PART III, INDICATE HERE:

EXPLAIN _____

APPENDIX C: PROTOCOLS FOR THE REMEMBRANCE OF
COMBINATORIAL LOGIC STRUCTURE

Arrangement of Geometric Figures
For Logical Memory Task



Protocol For Logical Memory Task--Immediate

Please examine the figures on this card. Describe what you see. Do you think it's all mixed up or is there some order to it? Do you notice anything else about the model? Do you care to make any other comments about it? Now try to fix the model in your mind because we are going to ask you to draw it from memory.

(Let student examine the card for as long as he wishes. Then put the card out of sight and give him a prepared form. The interviewer should write the name of the subject, the date, and his own name on the top of this form. The subject should put his memory drawing on this sheet.)

Please draw the model you have just seen as carefully and accurately as you can.

Protocol For Logical Memory Task--One Week

Please print your name on the top of the form. Write the date in the right corner of the paper. Indicate male or female with a check.

(Proctor should allow time for all students to complete the above directions before proceeding. Students should be quiet and attentive before reading the directions. The proctor may repeat the directions if requested by a student.)

Try to remember a white card you were shown last week. It contained drawings of geometric figures. You were asked to describe the figures on the card and whether or not the figures were mixed up. You tried to fix the model in your mind and then made a drawing of the geometric figures from memory. Please draw that model from memory

as carefully and accurately as you can. When you have completed the drawing turn your paper over and remain silent. Thank you.

Protocol For Logical Memory Task--Six Months

Geometric Figures--Drawing

(The interviewer should print the name of the subject in the upper left-hand corner of the form. Please indicate the date and your name in the upper right-hand corner).

Try to remember a white card you were shown at the beginning of the school year. It contained drawings of geometric figures. You were asked to describe the figures on the card and whether or not the figures were mixed up. You tried to fix the model in your mind and then made a drawing of the geometric figures from memory. Would you please state in a few words what you remember of that model?

(Please allow student time to respond. You may repeat the directions if requested by the student. You may attempt to prompt the student if necessary but do not give the subject any specific information about the model).

Please draw that model from memory as carefully and accurately as you can.

Protocol For Logical Memory Task
Reconstruction (Six Months)

Geometric Figures--Reconstruction

Now I would like you to reconstruct the model of the geometric figures as you remember it. You may use as

many of the cardboard figures as you wish but you do not have to use all of them unless you feel it is required. Please place the cardboard figures on the white card in exactly the same position as they appeared on the white card that you were shown at the beginning of the school year.

(When the student has completed the reconstruction, the interviewer should print the name of the student in the upper left-hand corner. The interviewer should mark the top of the form "R" and copy the reconstruction of the student).

Protocol and Score Sheet For Logical Memory
Six Month Recognition

NAME _____ DATE _____
 CLINICIAN _____

(Place the diagrams in front of the student so that they are all visible at once. Use a different order for each subject).

Please select the picture of the geometric figures that most closely resembles the model you were shown earlier in the school year.

(Circle One)

Choice	A	B	C	D	Comments
--------	---	---	---	---	----------

Probe--Are you sure this is the one that most closely resembles the one you were shown earlier?

YES _____ NO _____ Undecided _____

Comments (New Choice)

Is the one you chose an exact copy of the model you saw earlier or does it just resemble it?

Exact _____	Similar _____	Comments
-------------	---------------	----------

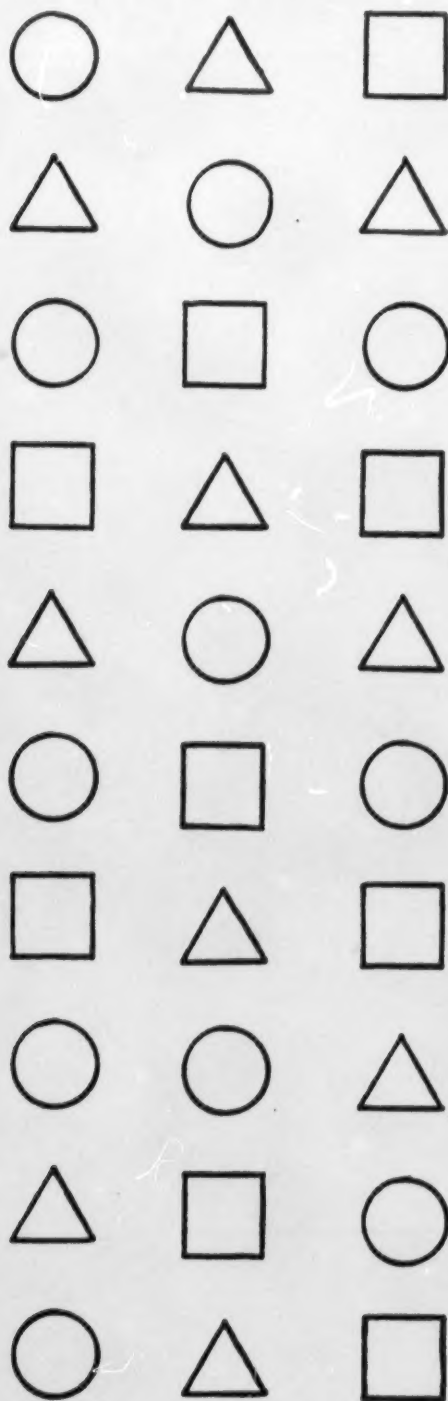
Diagram A for Six Month Recognition

Diagram B for Six Month Recognition

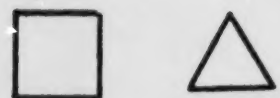
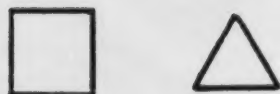


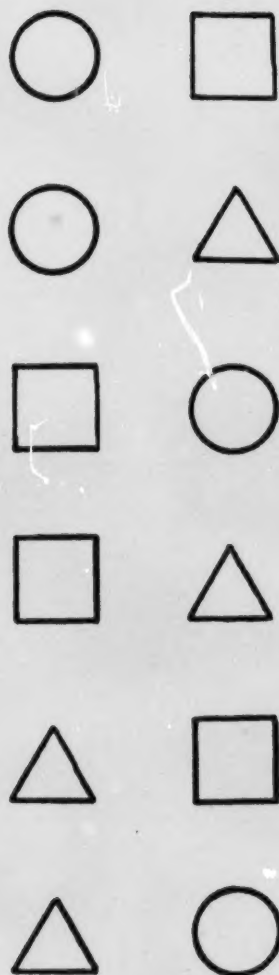
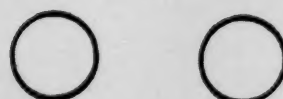
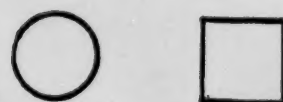
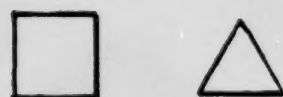
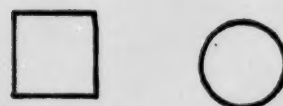
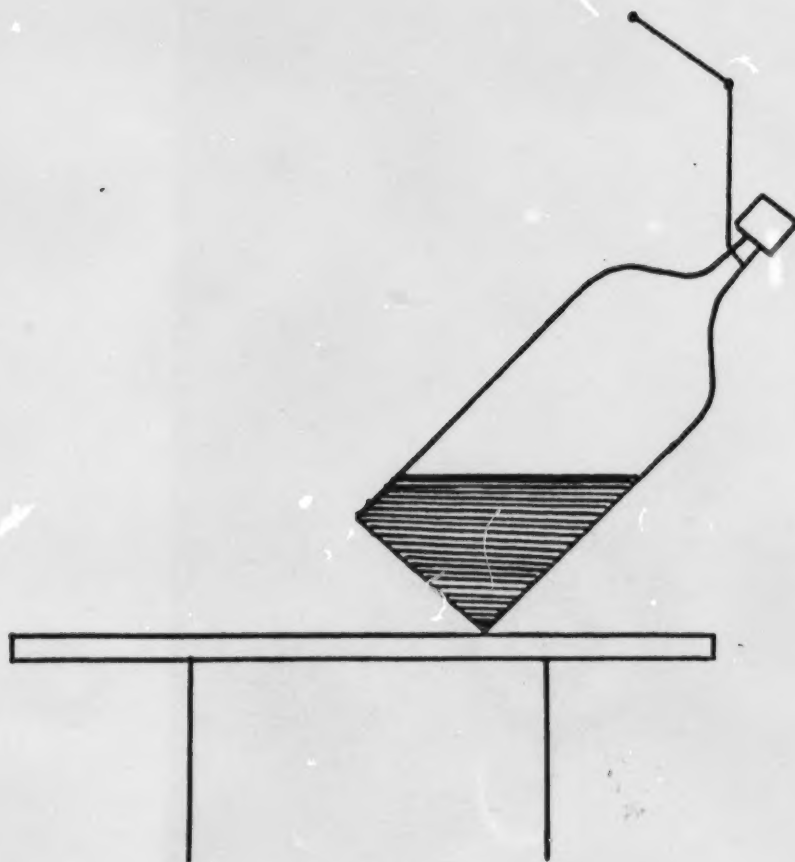
Diagram C for Six Month Recognition

Diagram D for Six Month Recognition



APPENDIX D: PROTOCOLS FOR THE
INCLINED BOTTLE TASK--SPATIAL MEMORY

Inclined Bottle Task Configuration



Protocol for Inclined Bottle Task
Immediate Memory

Please examine the figure on this card and try to fix the model in your mind because we are going to ask you to draw it from memory.

(Let student examine the card as long as he wishes. Then put the card out of sight and give him a prepared form. The interviewer should write the name of the subject, the date, and his own name on the top of this form. The subject should put his memory drawing on this sheet).

Please draw the model you have seen as carefully and accurately as you can.

Protocol for Inclined Bottle Task
One Week Memory

Please print your name on the top of the form. Write the date in the upper right corner of the paper. Indicate male or female with a check.

(Proctor should allow time for all students to complete the above directions before proceeding. Students should be quiet and attentive before reading the directions. The proctor may repeat the directions if requested by a student).

Try to remember a white card you were shown last week. It contained the drawing of a bottle with some red liquid in it. Please draw that model from memory as carefully and accurately as you can. When you have completed the drawing turn your paper over and remain silent. Thank you.

Protocol for Inclined Bottle Task
Six Month Memory

Bottle--Drawing

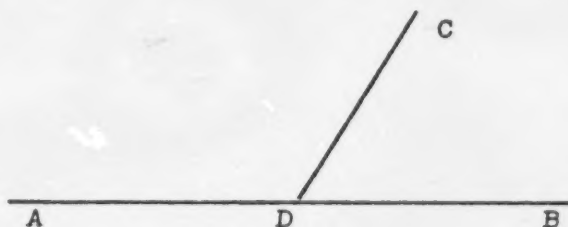
(The interviewer should print the name of the subject in the upper left-hand corner of the form. Please indicate the date and your name in the upper right-hand corner).

Try to remember a white card you were shown at the beginning of the school year. It contained the drawing of a bottle with some red liquid in it. Please draw that model from memory as carefully and accurately as you can.

APPENDIX E: PROTOCOLS FOR THE ANGLE MEASUREMENT
AND SPATIAL OPERATIVE ASSESSMENT TASKS

Protocol for Angle Measurement Task

- A) Subject is shown a drawing of two supplementary, adjacent angles ($\angle ADC$, $\angle CDB$)--place one drawing on table behind subject--place empty, white paper in front of subject.



- B) Subject may study the drawing and refer to it as often as subject likes but is NOT permitted to look at drawing while subject attempts to draw--encourage the use of tools but do NOT insist and say -

"Would you draw an exact copy of this on the blank paper. You may look at the drawing as often as you please--but you may not pick it up and look at it while you draw."

Angle Measurement Score Sheet

Name _____ Date _____

Sex _____ Interviewer _____

COMMENTS

1) Purely visual _____

2) Measures segments using _____

a) AB _____

b) AD _____

c) DB _____

d) DC _____

e) AC _____

f) CB _____

g) CX _____ (Perpendicular
distance)

h) DX, BX _____

3) Slope

a) Purely visual _____

b) Maintains slope using _____

c) Slope fixed via operations _____

Protocol and Score Sheet for Six MonthSpatial Operational Assessment

Name _____ Date _____

Clinician _____

Please examine the drawing on this card. Is it possible for the liquid to assume the level drawn on the card?

I.	(2)	Yes	No	Explanation (Why or Why Not)	Comments
----	-----	-----	----	---------------------------------	----------

Probe (Could the liquid be in a different position? Are you sure? Is this right now?)

	Yes	No	Explanation (Why or Why Not)	Comments
--	-----	----	---------------------------------	----------

II.	(3)	Yes	No	Explanation (Why or Why Not)	Comments
-----	-----	-----	----	---------------------------------	----------

Probe (See above)

	Yes	No	Explanation (Why or Why Not)	Comments
--	-----	----	---------------------------------	----------

III.	(6)	Yes	No	Explanation (Why or Why Not)	Comments
------	-----	-----	----	---------------------------------	----------

Probe (See Above)

	Yes	No	Explanation (Why or Why Not)	Comments
--	-----	----	---------------------------------	----------

NOTE: Please show the cards to each student in a different order. However, be sure to record the response in the correct place on the form. (i.e., the student's response for card 6 is last on the form regardless of its order in the presentation.

Diagram 2--Spatial Operational Assessment

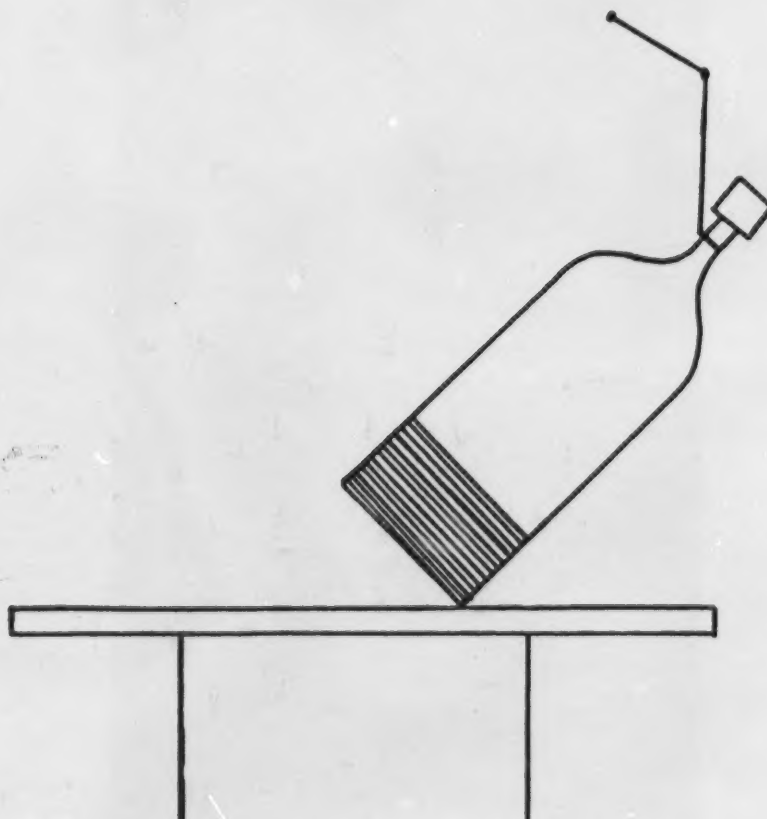


Diagram 3--Spatial Operational Assessment

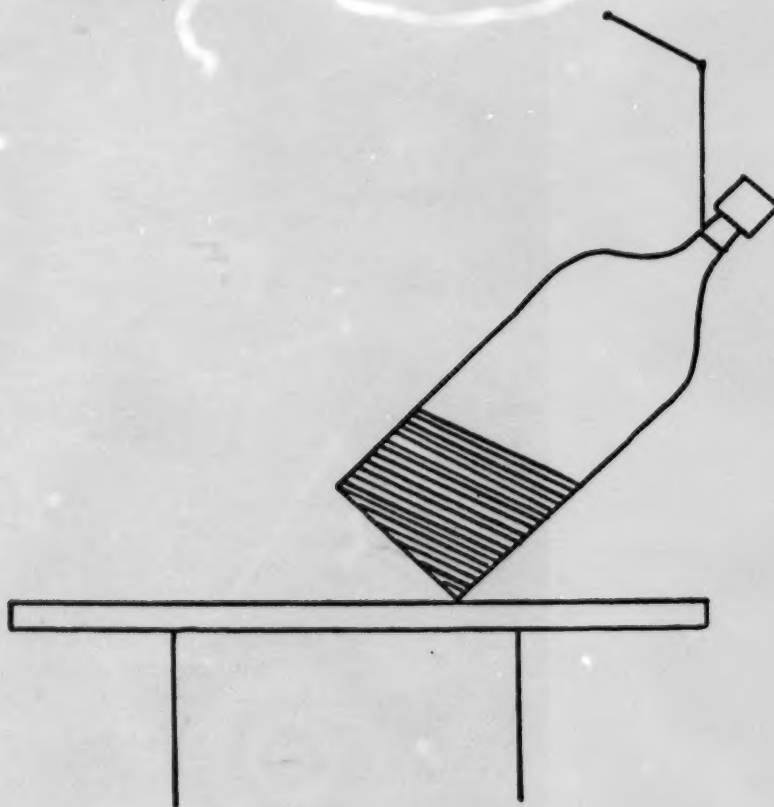


Diagram 6--Spatial Operational Assessment

